TEACHING TO SOLVE GEOMETRIC PROBLEMS USING THE METHOD OF VECTORS AND COORDINATES

[1]Prenov Barlikbay Barakbaevich, [2]Oteniyazov Erkin Turganbaevich
[1]Doctor of physical and Mathematical Sciences of the Department of methods of teaching mathematics of Nukus State Pedagogical Institute, Docent
[2]Assistant of the Department of methods of teaching mathematics of Nukus State Pedagogical Institute
DOI: 10.47750/pnr.2023.14.03.090

Abstract

This article presents the interests of bólgan in solving geometric problems in students, in which they are given instructions to strengthen the bólgan kónikma to solve the problem by solving the examples given during class hours using coordinates and vectors to bring this process to the surface.

Keywords: mathematics, methodology, method of coordinates, vector, section, point, middle line, bisector, theorem of cosines, surface.

INTRODUCTION

Currently, in our Republic, the main task is to prepare students with high intellectual potential, modern knowledge and world view who can think independently in general secondary educational institutions. Of course, in order to achieve such results, it is necessary for mathematics teachers to introduce a new approach to education, develop advanced pedagogical technologies, create and maintain modern educational and methodological complexes. In this process, the teacher leads to the development of the world view of the students by knowing the unique aspects of the academic subject, the innovations in the subject and being able to convey them to the students.

For this, the teacher must first of all have the following components:

High-level theoretical, practical and methodical training in one's specialty

Knowledge of modern methodical, pedagogical and information technologies

Having the skills of self-analysis and evaluation.

In order to more effectively organize mathematics lessons in general education schools, modern approaches and changes are demanded in the educational programs. According to the curricula approved by the Ministry of Public Education, geometry lessons in 8-9 grades include topics related to triangles, rectangles, and circles. To strengthen the topic, part of the lesson hours is divided into repetition hours.

When students enter a new class, it becomes a little more difficult to memorize the topics covered. As a result, the student may become lazy or not interested in the lesson. In order to prevent this process, the teacher shows students how to solve a previously studied problem or theorem with a different new method during repetitive lessons, and gives them tasks and strengthens the students' interest in science. Therefore, as a result of this, a world of new knowledge emerges in the student's mind through logical thinking and observation of properties that have been proven to be correct. Knowing the dry theory is not enough to solve mathematical problems. Problem solving skills and experience are also required. The main task of repetitive lesson hours is to be organized.

By solving problems in a number of ways in geometry lessons in general education schools, students' problem-solving skills and experience begin to increase. In order to have this kind of knowledge, starting from simple problems and solving complex problems, mastering different methods of solving problems and being able to retain it is achieved. In this process, the teacher should be able to create an environment in which students are interested in science and acquire new knowledge through their high pedagogical skills. In geometry lessons, we show that problems related to triangles, rectangles, and circles are usually solved using formulas related to this topic.

Algebraic, vector, surface method, coordinate, similarity, and geometric replacement methods are widely used in the school course for solving geometric problems. If we teach solving using the vector and coordinate method, which is one of the above methods, the skills described above will begin to develop. In the educational process, teaching the vector and coordinate method and forming the skills of solving problems based on this method is not only important for the formation of knowledge and skills in students, but also for the formation of the skills to apply this method to solve problems related to metric problems in plane and space.

For this, the students should have a complete understanding of the concept of vector, addition, subtraction and multiplication on vectors, operations on the coordinates of vectors, scalar multiplication of vectors, the geometric position of points in the coordinate system, the distance between points and formulas for finding the coordinates of their location. When solving problems using the above methods, it is appropriate to work on the basis of the following algorithm.

Analyzing the content of the problem and translating it into the language of vectors and coordinates;

Using known properties, create equations and find equations;

Changing the form of expressions and calculating the value;

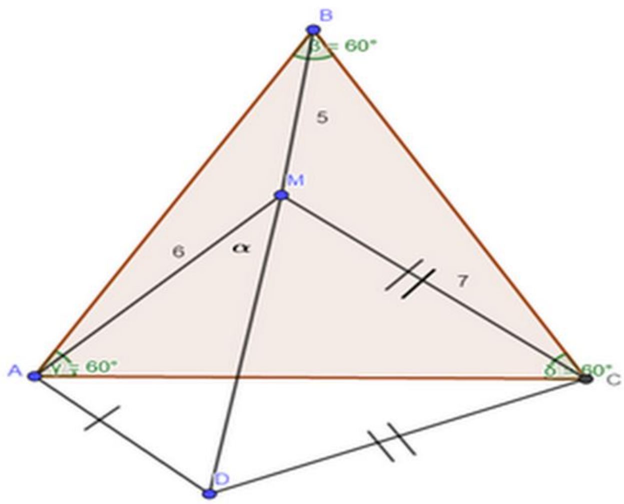
Converting the result into a geometric language;

Write the answer.

The quality of teaching will be improved if we show students how to solve the problems given during the lesson using the geometric, vector, coordinate method. We will consider how to solve some issues related to school textbooks in several ways.

Example 1. The distances from the interior point to the vertices of a regular triangle are equal to 5, 6 and 7. Find the face of the given triangle.

We solve the given problem using the properties of the triangle as follows:



According to the condition of the matter.

We turn the sphere around the end C, and the sides are congruent. If the image of the point falls on the point, it corresponds to the section , and to and connect the points and study the sides of the resulting triangle (Fig. 1). -equal sided and the angle between them follows from the fact that it is regular. So, all sides.

So we can find the theorem of cosines.

According to the condition of the matter, the triangle $MB=5, MA=6,$ turns around the C end up to 60^0 , here CB and CA sides are congruent. If the image of the point M falls on the point D , for CD line CM , for BM , AD corresponds. $([5])$.

M and D , correspond. .

and connect the points and study the sides of the resulting triangle (Fig. 1). ΔCMD -equilateral $(CM=CD)\ 60^0$ from being comes the fact that it is regular. So, ΔAMD all sides $MD=7, AD=5, MA=5$.

ΔAMD cosines for the theorem support $\alpha=\angle AMD$ topamiz.

$$\cos\alpha=\frac{36+49-25}{2\cdot6\cdot7}=\frac{5}{7},\ \ \sin\alpha=\sqrt{1-\frac{25}{49}}=\frac{2\sqrt{6}}{7}\ \ (\alpha<90^0).$$

$\angle AMC\ \ 60^0+\alpha$ we will be equal and we will solve the formula of the sum of cosines to find its value.

$$\cos\left(60^0+\alpha\right)=\frac{1}{2}\cdot\frac{5}{7}-\frac{\sqrt{3}}{2}\cdot\frac{2\sqrt{6}}{7}=\frac{5-6\sqrt{2}}{14}.$$

Now for the cosines we find the side, applying the theorem.

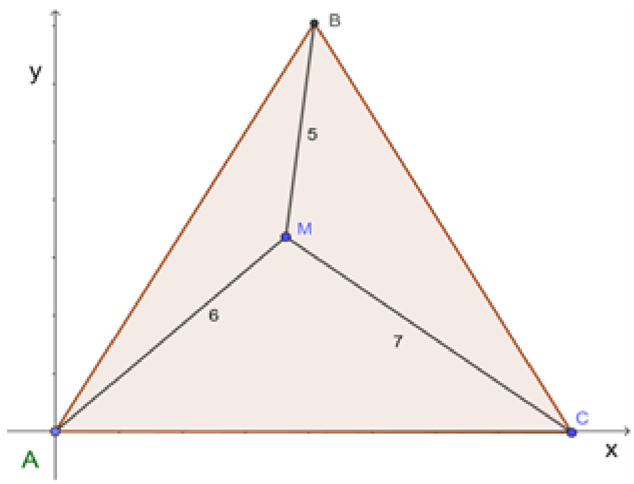
$$AC^2=36+49-2\cdot6\cdot7\cdot\frac{5-6\sqrt{2}}{14}=55+36\sqrt{2}$$

One side of a regular Triangle has been found, now let's calculate it through the formula for finding the surface..

$$S_{ABC}=\frac{AC^2\sqrt{3}}{4}=\frac{55\sqrt{3}+36\sqrt{6}}{4}.$$

Of course being able to apply the above steps may seem a little more complicated to the reader. But if we solve a given problem with the coordinate method, we will look more simplistic.

If so that $AB=BC=AC=a,$ and $A(0;0)$ we place the point at the beginning of the coordinates. The abscissa axis has the side, while the ordinate axis is perpendicular to it. We place the points as follows: (Figure 2)



(2-fig)

$$C(a;0),\ B\left(\frac{a}{2};\frac{a\sqrt{3}}{2}\right), M(x;y)$$

According to the condition of the issue $AM=6, CM=7$.

$$\begin{cases}x^2+y^2=AM^2\\(x-a)^2+y^2=CM^2\end{cases}\Rightarrow\begin{cases}x^2+y^2=36\\(x-a)^2+y^2=49\end{cases}\Leftrightarrow\begin{cases}x^2+y^2=36\\x^2-2ax+a^2+y^2=49\end{cases}$$

From the second equality in the system $x=\frac{a^2-13}{2a}$. so BM we draw up an equation that represents the intersection of.

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{a\sqrt{3}}{2}\right)^2=25$$

$$x^2-ax+\frac{a^2}{4}+y^2-ay\sqrt{3}+\frac{3a^2}{4}=25$$

$x^2 + y^2 = 36$ we use equality.

$11 + a^2 = ax + ay\sqrt{3}$, y find it $y = \dfrac{a^2 + 11 - ax}{a\sqrt{3}} = \dfrac{a^2 + 11 - \dfrac{a^2 - 13}{2}}{a\sqrt{3}} = \dfrac{a^2 + 35}{2a\sqrt{3}}$.

So M point coordinates $M\left(\dfrac{a^2 - 13}{2a}; \dfrac{a^2 + 35}{2a\sqrt{3}}\right)$. M coordinates of the point a expressed using the parameter. We will use the equation of the Decimal Fraction to find the side of the triangle again.

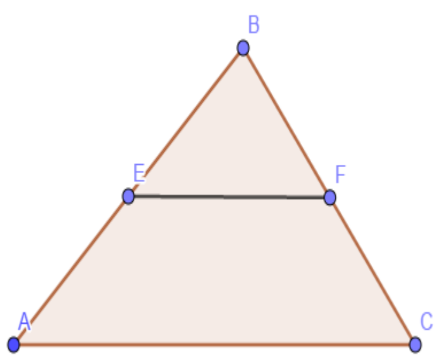
$x^2 + y^2 = 36$, $\dfrac{a^4 + 70a^2 + 1225}{4a^2 \cdot 3} + \dfrac{a^4 - 26a^2 + 169}{4a^2} = 36$, $a^4 - 110a^2 + 433 = 0$, $a^2 = 55 \pm 36\sqrt{2}$. The Triangle AC square of the side $a^2 = 55 + 36\sqrt{2}$ will be equal. Using the formula for calculating the surface, $S_{ABC} = \dfrac{\sqrt{3}}{4} AC^2 = \dfrac{55\sqrt{3} + 36\sqrt{2}}{4}$ comes from.

Example 2. The center line of the Triangle is parallel to its third side, the length of which is half the length of this side. Prove the theorem.

Solution: students are taught to prove the theorem using the 8th grade geometry textbook Thales theorem, but few students understand this method. In order for this process to be consequential, we will make it a task for students to prove the theorem in the language of vectors, like after the subject of the vector, during class hours. As a result of this, the previous proof of the theorem is brought to mind or, using a teacher, then proceed to proof by vectors $([10])$.

We cite the following.

- - Triangle
- EF - ABC middle line of the Triangle
- $EF = \dfrac{1}{2}AC,\ EF \parallel AC$ must be proven.



3-fig

We translate a given problem into vector language. $EF \parallel AC$, so $\overrightarrow{EF}$ and $\overrightarrow{AC}$ vectors collinear vectors, that $\overrightarrow{EF} = k\overrightarrow{AC}$. $k = \dfrac{1}{2}$ so, $\overrightarrow{EF} = \dfrac{1}{2}\overrightarrow{AC}$ we must prove that it is. (Figure 3)

When solving the problem, we use the triangular rule of dividing vectors.

- $\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF}$

- $\overrightarrow{EB}=\frac{1}{2}\overrightarrow{AB},\ \overrightarrow{BF}=\frac{1}{2}\overrightarrow{BC}$

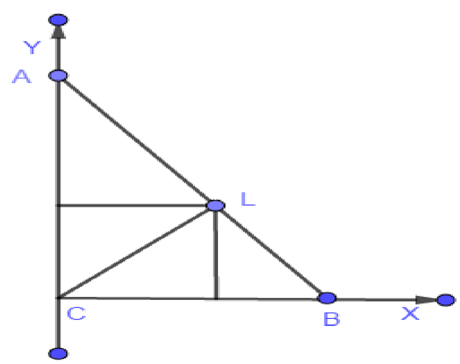
- We add the above

- $\overrightarrow{EF}=\frac{1}{2}\left(\overrightarrow{AB}+\overrightarrow{BC}\right)=\frac{1}{2}\overrightarrow{AC}$. So $EF \square AC$ from being $EF=\frac{1}{2}AC$.

Example 3. Cathets of a triangle with a right angle $3:4$ in proportion. Find the perimeter of a triangle if the length of the bisector of a right angle is equal.

The given issue is given in the 8th grade geometry textbook, we will consider its solution not according to the textbook, but using methods on the collar.

Coordinate method: Triangle $A(0,4t), B(3t,0), C(0,0)$ we place in points and $L(x,y)$ bissect AB we mark the point on the side. L point coordinates are coordinates that pass through the head $x=y$ is determined by the equation $([4])$. (4-fig)



4-fig

AB we draw up the equation of the line of the transitive circle by $y=-\frac{4}{3}x+4t$. L AB as belonging to the party,

$L(24,24)$ point AB by $24=-\frac{4}{3}\cdot 24+4t,\ t=14.$

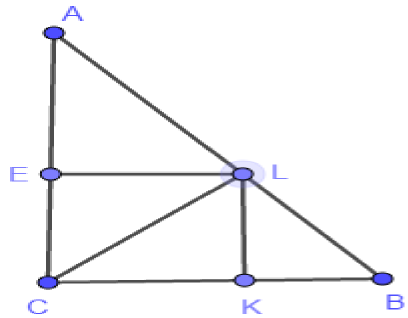
so $P=12t,\ P=168.$

Now we solve using vectors. Point by point condition $AL:LB=4:3$, in it, through the formula for adding vectors $\overrightarrow{CL}=\frac{3}{7}\overrightarrow{CA}+\frac{4}{7}\overrightarrow{CB}$ dan $([1],[4])$ we use equality. This formula comes from:

Bissektrisa $\overrightarrow{CL}=\overrightarrow{EL}+\overrightarrow{LK}$ by equality we find. $\overrightarrow{AC}$ and $\overrightarrow{LK}$, $\overrightarrow{CB}$ and $\overrightarrow{EL}$ from the fact that vectors are collinear

$$\overrightarrow{LK}=\lambda_1\overrightarrow{AC},\ \overrightarrow{EL}=\lambda_2\overrightarrow{CB}$$

so $\overrightarrow{CL}=\lambda_1\overrightarrow{AC}+\lambda_2\overrightarrow{CB}$. (5-fig)



(5-fig)

Now using the similarity properties of the Triangle λ_1, λ_2 we express the bissectrisa through the proportions of being.

$$\lambda_1 = \frac{\overrightarrow{LK}}{\overrightarrow{AC}} = \frac{\overrightarrow{LB}}{\overrightarrow{AB}} = \frac{n}{m+n}, \ \lambda_2 = \frac{\overrightarrow{EL}}{\overrightarrow{CB}} = \frac{\overrightarrow{AX}}{\overrightarrow{AB}} = \frac{m}{m+n}$$

$$\overrightarrow{CL} = \lambda_1 \overrightarrow{AC} + \lambda_2 \overrightarrow{CB} = \frac{n}{m+n} \overrightarrow{AC} + \frac{m}{m+n} \overrightarrow{CB}.$$

In here results

$$\overrightarrow{CL} = \frac{3}{7}\overrightarrow{CA} + \frac{4}{7}\overrightarrow{CB}$$

resolve.

Latest PCB $\overrightarrow{CL}^2 = \frac{9}{49}\overrightarrow{CA}^2 + \frac{16}{49}\overrightarrow{CB}^2 + \frac{12}{49}\overrightarrow{CA}\cdot\overrightarrow{CB}$.

$$24^2 \cdot 2 = \frac{9}{49} \cdot 16k^2 + \frac{16}{49} \cdot 9k^2 + 0, \ k = 14$$

$$P = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AC} = 12k, \ P = 168.$$

Of course, this is a very complex process, but it is the bólib that takes great steps to reach the higher class of teachers, scientists, doctors of science, through the knowledge that has been applied with the help of such research.

In school practice, the coordinate and Vector method is much less. The essence as a method of solving problems in similar methods is that by constructing numbers with equations and expressing various geometric relationships in coordinates, we can solve a geometric problem using algebra. Therefore, if we consider that the given problems in science are solved using different methods, we can see that the tópalized knowledge of the óqar is more robust.

REFERENCES

1. I.Kushnir. Vector methods for solving problems. Publishing house "Oberig", Kiev, 1994.
2. V.S.Bolodurin. Coordinate-vector method for solving geometric problems. Methodical manual for teachers of mathematics. Orenburg OGPU Publishing House 2015.
3. E.V.Potoskuyev. Vector-coordinate method for solving stereometry problems. Exam Publishing House, Moscow 2019.
4. O.P.Zelenyak. Solving problems on planimetry. The technology of an algorithmic approach based on problem-theorems. Kiev, Moscow: Diasoftyup, DMK Press, 2008.- 336 p.
5. N.D.Dodaconov, M. Sh.Jóraev. "geometry" Part 1. Educational guide for higher educational institutions. 2nd edition.- T.: Teacher, 1996-384 P.
6. S.Alikhanov. Publishing and printing creative house named after Chulpan" methods of teaching mathematics". Tashkent, 2011, -304 P.
7. Rahimqoriyev A.A, M.A.Takhtakhodjaeva. Textbook for the 8th grade of general secondary schools" geometry " T. "Uzbekistan" 2019.-160 p.
8. B.Haydarov, E.Sarigov, A.Koshkorov. Textbook for the 9th grade of general secondary schools" geometry". Tashkent 2019.-160 p.

9. B.Haydarov. Textbook for the 10th grade of general secondary schools" geometry". Tashkent 2017.-144 p.
10. Prenov B. B., Nurmakhanov K. E. Active methods of teaching analytical geometry in higher education institutions //NeuroQuantology | November 2022 | Volume 20 | Issue 18 | Page 108-111 | doi: 10.14704/nq.2022.20.18.NQ880013.
11. K.E.Nurmakhanov. Important factors for using the geogebra program in teaching analytical geometry //Galaxy international interdisciplinary research journal (GIIRJ) ISSN (E): 2347-6915 Vol. 10, Issue 11, Nov. (2022).
13. K.E Nurmakhanov. Improving the Methodology of Using the Geogebra Program in E-Learning Environment //Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 14571 - 14577