A Mathematical Analysis on the Dynamics of Macrophytes in Varthur lake, Bangalore

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Abstract

In this paper, a case study is presented to study the effects of increasing nutrients and macrophytes on survival of fish population in Varthur lake, Bangalore, India. Several fish kill episodes have occurred in Varthur lake over past years. Aim of the present study is to investigate the effects mathematically as huge amount of nutrients from industries, domestic sewage is constantly release into the lake which is leading to abundant growth of macrophytes leaving less room for fish survival. The above-mentioned parameters are considered to formulate a mathematical model. Equilibrium points are studied and analysed in detail for stability via Routh-Hurwitz criteria. Numerical simulation is carried out to illustrate the analytical findings of the case study.

Keywords: Nutrients, Macrophytes, Fish Population, Stability Analysis, Routh-Hurwitz Criteria.

INTRODUCTION

Lakes provides immerse source of biodiversity. Several species depend on healthy lakes for their survival. They contribute to a major percentage of fresh water which is fit for drinking, meeting domestic and agricultural needs, unfortunately the fresh water ecosystem facing a threat due to increase in population, climate change, human activities leading to pollution and lake encroachment. Lake pollution is worsening over the past few years, as they are most vulnerable to pollution due to their easy accessibility for the disposal of pollutants and wastewater [1]. Waste from industries, domestic sewage, agricultural run offs are major sources of pollution in lakes. These pollutants carry excessive nutrients promote massive plant growth. Most of the lakes/ponds are observed to be under considerable threat due to excessive growth macrophytes [2, 3] which affects the species residing in it. Bangalore, which was once known as the city of lakes, has lost several lakes due to unplanned rapid urbanization, IT Boom in the past few years leaving no space for healthy lakes. The natural resources are facing major deterioration issues due to the unthoughtful plundering by the people [4].

Due to rapid urbanization, the demand for water and land has drastically increased. More recently, housing has caught upon this growth and vast areas of the city that was once green has now been converted to real estate and housing [5]. The lakes are encroached, domestic and industrial waste has been dumped into the them. As a result, several lakes of Bangalore such as Bellandur lake, Varthur lake, Sarakki lake, Hebbal lake, Ulsoor lake etc., are facing massive fish kill, highly polluted, frothing and catching fire. The city is also facing many levels of water stress such as pollution of lakes, flooding of residential area during heavy rains due to poor infrastructure and poor water management [6].

In this paper, a case study of Varthur lake is considered. Lake pollution problem has attracted many researchers and several models have been studies and analysed to solve the problem. Macrophytes also play a vital role in functioning of an aquatic ecosystem. Studies about excessive aquatic macrophyte growth and control have long been an issue everywhere [7]. Some of the macrophytes studies with respect to Varthur lakes are as follows. In [8] authors have studied the dynamics of metal pollution and growth of macrophytes in Varthur lake. Macrophytes also play a vital role in structuring and functioning of an aquatic ecosystem. The role and diversity of macrophytes in Varthur and Bellandur lake is showcased in the paper [9]. The nutrient enrichment and the contribution of macrophytes to urban lakes is studied in [10]. The lower rate of dissolved oxygen in the lake was due to the excessive growth of macrophytes as it prevented light from penetrating into water which resulted in low rates of photosynthesis by phytoplankton. In [11] a case study for Varthur lake is considered where the study concluded that the lake has high concentrations of phosphorous and organic matter and it is eutrophic. In [12] a case study for Varthur lake is proposed to evaluate heavy metal contamination. The paper [13] exhibits the analysis of water quality and its environmental impact on Varthur lake. The sample from the lake was collected and a detailed physio-chemical parameter analysis was conducted. A Technical report is presented [14] for Bellandur and Varthur Lakes Rejuvenation, wherein the authors present a detailed discussion on consequences of pollution, heavy metals, frothing, sewage pollution plants, waste water treatment process etc. The authors in [15] modelled the pollution in a system of lakes. Here, a system of three lakes is introduced which are interconnected by channels flowing between them. Further, the model is mathematically analysed with three different types of input model to monitor pollution. In [16] authors studied the Spatio-Temporal Dynamics of Algae and Macrophyte Cover in Urban Lakes using remote sensing analysis.

Researchers have also studied various polluted lakes across Bangalore city [17,18,19]. In [20], authors have considered a study on Ulsoor lake in Bangalore. The increasing human population effects on the survival of fish in Ulsoor lake directly by fishing and indirectly by human activities in polluting the lake is portrayed by the study. The paper also suggests control on additional pollutants entering the lake to maintain the survival of fish population. The authors presented a detailed study [21] on accumulation of heavy metals in Bellandur lake, the results showed that water samples contained heavy metals throughout the study area. The metal concentration in fish tissue was also evaluated.

In view of above described, a mathematical model is proposed to study the effects of nutrients and macrophytes on fish population in Varthur lake. The paper is sectioned as follows: Section 2 presents a detailed case study on Varthur lake. A mathematical model is formulated in section 3. In Section 4, the equilibrium points of the system of equations are shown. Section 5 describes the stability of the system of equations. In section 6, Numerical simulation is performed and lastly the work is concluded.

CASE STUDY: VARTHUR LAKE

Varthur lake is the second largest lake at 12° 57′ 09″ N and 77° 44′ 33″ E in Bangalore city according to Bruhat Bengaluru Mahanagara Palike (BBMP). According to ENVIS Technical Report: 116, Varthur Lake has an area of nearly 279 square kilometres with 96 cascading interlinked lakes [14]. A drastic increase in population in the city resulted in decline of vegetation. The lake surrounding is densely populated and agricultural land, water bodies, open land is encroached by huge built-up area in the recent few years. Untreated sewage waste, industrial effluents are directly released into the lake which resulted in fish mortality, decline in ground water and frothing of lakes. Several episodes of fish kill are reported in the articles [22,23,24,25]. According to the study the lake has higher concentration of nutrients which results in algal bloom and spread of macrophytes. Considering these (Nutrients, Macrophytes, Fish) parameters a mathematical model comprising the system of non-linear ordinary differential equation is proposed to study the present state of Varthur lake.

Figure 1: Satellite image of Varthur lake modified from mapcarta

Figure 2: Images from the site – Varthur lake (a)Pollutants released at Varthur lake (b) Free floating macrophytes covered at Varthur lake.

MATHEMATICAL MODEL

In this section, a mathematical model is formulated considering three parameters. Where, is considered to be the concentration of nutrients, is considered to be the density of macrophytes and is considered to be the fish population. It is assumed that there is cumulative rate of discharge of nutrients into the water body at constant rate [26]. Nutrients deplete naturally at the rate of , Nutrients are taken up by macrophytes at the rate of . Macrophytes deplete naturally at the rate of , Macrophytes are consumed by fish population at the rate of and is the natural depletion rate of fish population. Here, , and are depletion coefficients and all the parameters are taken to be positive constants. Considering these parameters, the mathematical model is formulated as follows:

$$\frac{dN}{dt} = I - b_0 N - a_1 NM$$

$$\frac{dM}{dt} = \gamma_1 a_1 NM - b_1 M - a_2 MF$$

$$\frac{dF}{dt} = \gamma_2 a_2 MF - b_2 F \qquad (1)$$

Where N(0) > 0, M(0)>0, F(0) > 0

Table 1: Parameters and its values

Parameter	Value
I -Cumulative discharge of nutrients (nitrogen, phosphorous) into the water body at a constant rate	10
b_0 – rate of nutrients loss	1.6
b_1 – Natural depletion of macrophytes	1.2
b_2 - Fish mortality rate	3.1
γ_1 – growth rate of macrophytes due to nutrients	5.5
γ_2 - growth rate of fish population due to macrophytes	1.7
a_1 – depletion rate of Nutrients by macrophytes	0.1
a_2 – depletion rate of macrophytes by fish	0.2

EQUILIBRIUM POINTS OF THE SYSTEM

The equilibrium points of the model (1) can be obtained by setting $\frac{dN}{dt} = 0, \frac{dM}{dt} = 0,$ and $\frac{dF}{dt} = 0$.

The system produces three dynamic equilibrium points $E_i = (N, \mathrm{M}, F)$ where $i = 1, 2, 3.$

Here, $E_1 = (\frac{I}{b_0}$, 0, 0) always exists and initially, it is considered that there is no change in the rate of flow of nutrients and the fish population is negligibly small,

$E_2 = \left(\frac{b_1}{a_1\gamma_1}, \frac{Ia_1\gamma_1 - b_0 b_1}{b_1 a_1}, 0 \right)$ and

$$E_3 = \left(\frac{Ia_2\gamma_2}{a_2 b_0 \gamma_2 + a_1 b_2}, \frac{b_2}{a_2\gamma_2}, \frac{Ia_1 a_2 \gamma_1 \gamma_2 - a_2 b_0 b_1 \gamma_2 - b_2 b_1 a_1}{(a_2 b_0 \gamma_2 + a_1 b_2) a_2} \right)$$

STABILITY OF THE SYSTEM

The variational matrix J of the system (1) is given by,

$$J = \begin{bmatrix} -Ma_1 - b_0 & -a_1 N & 0 \\ Ma_1\gamma_1 & Na_1\gamma_1 - Fa_2 - b_1 & -a_2 M \\ 0 & Fa_2\gamma_2 & Ma_2\gamma_2 - b_2 \end{bmatrix}$$

(2)

Theorem 1: E_1 is said to be locally asymptotically stable (LAS), if the eigenvalues obtained satisfy the Routh–Hurwitz Stability Criteria, otherwise unstable.

Proof: Corresponding to the equilibrium point $E_1 = (\frac{I}{b_0}$, 0, 0), the Jacobian matrix J_1 is obtained below,

$$J_1 = \begin{bmatrix} -b_0 & \frac{-Ia_1}{b_0} & 0 \\ 0 & \frac{Ia_1\gamma_1}{b_0} - b_1 & 0 \\ 0 & 0 & -b_2 \end{bmatrix}$$

The eigen values obtained are $\lambda_1 = \frac{Ia_1\gamma_1 - b_0 b_1}{b_0}, \lambda_2 = -b_0, \lambda_3 = -b_2$. Clearly, the two eigenvalues of the matrix J_1 are negative and the other eigenvalue form the roots of a quadratic equation $\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$ (3)

Where,

$$A_1 = -\frac{I\gamma_1 a_1 - b_0^2 - b_1 b_0 - b_2 b_0}{b_0}$$

$$A_2 = -\frac{I\gamma_1 a_1 b_0 + Ia_1 b_2 \gamma_1 - b_1 b_0^2 - b_0^2 b_2 - b_0 b_1 b_2}{b_0}$$

$$A_3 = -b_2 (I\gamma_1 a_1 - b_0 b_1)$$

Based on Routh-Hurwitz criteria, all the eigen values obtained from the matrix J_1 must be negative. Here, it is noted that the two eigen values λ_2 and λ_3 are negative. Therefore, E_1 is unstable, it will be stable if and only if λ_1 is also negative $(\lambda_1 < 0)$.

Theorem 2: E_2 is said to be locally asymptotically stable (LAS), if the eigenvalues λ_i obtained satisfy the Routh–Hurwitz Stability Criteria, otherwise unstable.

Proof: Corresponding to the equilibrium $E_2 = \left(\frac{b_1}{a_1\gamma_1}, \frac{Ia_1\gamma_1 - b_0 b_1}{b_1 a_1}, 0 \right)$, the Jacobian matrix J_2 is obtained below,

$$J_2 = \begin{bmatrix} -\frac{I\gamma_1 a_1 - b_1 b_0}{b_1} - b_0 & -\frac{b_1}{\gamma_1} & 0 \\ \frac{(I\gamma_1 a_1 - b_1 b_0)\gamma_1}{b_1} & 0 & -\frac{a_2(I\gamma_1 a_1 - b_1 b_0)}{b_1 a_1} \\ 0 & 0 & \frac{(I\gamma_1 a_1 - b_1 b_0)a_2\gamma_2}{b_1 a_1} - b_0 \end{bmatrix}$$

Eigen values of the above matrix is given by,

$$\lambda_1 = \frac{Ia_1a_2\gamma_1\gamma_2 - a_2b_0b_1\gamma_2 - b_2b_1a_1}{b_1a_1},$$

$$\lambda_2 = -\frac{Ia_1\gamma_1 - \sqrt{-4Ia_1\gamma_1b_1^2 - a_1^2\gamma_1^2 + 4b_0b_1^3}}{2b_1},$$

$$\lambda_3 = -\frac{Ia_1\gamma_1 + \sqrt{-4Ia_1\gamma_1b_1^2 - a_1^2\gamma_1^2 + 4b_0b_1^3}}{2b_1}$$

Clearly, the two eigenvalues of the matrix J_2 are negative and the other eigenvalue form the roots of a quadratic equation

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$$

(4)

Where,

$$A_1 = \frac{-Ia_1a_2\gamma_1\gamma_2 + I\gamma_1a_1^2 + a_2b_0b_1\gamma_2 + b_2b_1a_1}{b_1a_1}$$

$$A_2 = \frac{Ia_2b_0b_1\gamma_1\gamma_2 + I\gamma_1a_1b_1^2 + Ia_1b_1b_2\gamma_1 + a_1a_2\gamma_1^2\gamma_1 - b_1^3b_0}{b_1^2}$$

$$A_3 = -\frac{(Ia_1a_2\gamma_1\gamma_2 - a_2b_0b_1\gamma_2 - b_2b_1a_1)(I\gamma_1a_1 - b_1b_0)}{b_1a_1}$$

Based on Routh-Hurwitz criteria, all the eigen values obtained from the matrix J_1 must be negative. Here, it is noted that the two eigen values λ_2 and λ_3 are negative. Therefore, E_2 is unstable, it will be stable if and only if λ_1 is also negative $(\lambda_1 < 0)$.

Theorem 3: E_3 is locally asymptotically stable with some conditions, otherwise it is unstable.

Proof: Now, corresponding to the coexistence equilibrium $E_3 = (N^*, \mathrm{M}^*, \mathrm{F}^*)$ where,

$$N^* = \frac{Ia_2\gamma_2}{a_2b_0\gamma_2 + a_1b_2}, M^* = \frac{b_2}{a_2\gamma_2}, F^* = \frac{Ia_1a_2\gamma_1\gamma_2 - a_2b_0b_1\gamma_2 - b_2b_1a_1}{(a_2b_0\gamma_2 + a_1b_2)a_2}$$

the Jacobian matrix J_3 is obtained below,

$$J_3 = \begin{bmatrix} -\frac{b_2 a_1}{a_2\gamma_2} - b_0 & \frac{-Ia_1a_2\gamma_2}{a_2b_0\gamma_2 + a_1b_2} & 0 \\ \frac{b_2a_1\gamma_1}{a_2\gamma_2} & \frac{Ia_1\gamma_1a_2\gamma_2}{a_2b_0\gamma_2 + a_1b_2} - \frac{Ia_1a_2\gamma_1\gamma_2 - a_2b_0b_1\gamma_2 - b_2b_1a_1}{a_2b_0\gamma_2 + a_1b_2} - b_1 & -\frac{a_2b_2}{a_2\gamma_2} \\ 0 & \frac{(Ia_1a_2\gamma_1\gamma_2 - a_2b_0b_1\gamma_2 - b_2b_1a_1)a_2\gamma_2}{(a_2b_0\gamma_2 + a_1b_2)a_2} & 0 \end{bmatrix}$$

Let,

$$m_1 = \frac{b_2a_1}{a_2\gamma_2} - b_0, m_2 = \frac{Ia_1a_2\gamma_2}{a_2b_0\gamma_2 + a_1b_2}, m_3 = \frac{b_2a_1\gamma_1}{a_2\gamma_2},$$

$$m_4 = \frac{Ia_1\gamma_1a_2\gamma_2}{a_2b_0\gamma_2 + a_1b_2} - \frac{Ia_1a_2\gamma_1\gamma_2 - a_2b_0b_1\gamma_2 - b_2b_1a_1}{a_2b_0\gamma_2 + a_1b_2} = 0,$$

$$m_5 = -\frac{a_2b_2}{a_2\gamma_2}, m_6 = \frac{(Ia_1a_2\gamma_1\gamma_2 - a_2b_0b_1\gamma_2 - b_2b_1a_1)a_2\gamma_2}{(a_2b_0\gamma_2 + a_1b_2)a_2}$$

Then, the Jacobian evaluated at the coexistence equilibrium point is given by,

$$J^*(N^*, M^*, F^*) = \begin{bmatrix} -m_1 & -m_2 & 0 \\ m_3 & 0 & m_5 \\ 0 & m_6 & 0 \end{bmatrix}$$

Taking, $\left|J^* - \lambda I\right| = 0$ in order to get the eigen values of the system.

The variational matrix below is obtained,

$$\left|J^* - \lambda I\right| = \begin{bmatrix} -m_1 - \lambda & -m_2 & 0 \\ m_3 & 0 - \lambda & m_5 \\ 0 & m_6 & 0 - \lambda \end{bmatrix} = 0$$

The characteristic equation of system at the coexistence equilibrium point $E_3 = (N^*, \mathrm{M}^*, \mathrm{F}^*)$ is $\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$ where, $A_1 = m_1, A_2 = m_1 - m_5m_6, A_3 = m_1m_5 - m_6$
(5)

From Routh–Hurwitz criterion, it follows that all the roots of the above equation have negative real parts provided the condition given below holds:

$$A_1 > 0, \begin{vmatrix} A_1 & 1 \\ A_3 & A_2 \end{vmatrix} = A_1A_2 - A_3 > 0$$

Thus, the roots of the Equation are negative or else they are having negative real parts if and only if the last inequality is satisfied [28]. Hence the coexistence equilibrium E_3 is locally asymptotically stable if A_i for $i = 1, 2, 3$ and $A_1A_2 - A_3 > 0$

NUMERICAL SIMULATION

In this section, MATLAB and Business Intelligence (BI) tool Tableau are used to analyse the data. The data has been collected from a project report for Bengaluru Development Authority (BDA) named Desilting Varthur Lake compiled by Sensing Local for Mineral Enterprises Ltd. (MEL). And couple of values has also been taken from [27] rest are assumed according to the model. The parameter and simulation values are shown in Table 1 and Table 2.

Figure 3 shows the long term (2002-2019) trend of macrophytes covering the surface of Varthur lake [16]. The figure clearly shows the increase in percentage of macrophytes which in turn affects the fish population residing in the lake. In Figure 4, a forecast is presented until 2023 using average the data from [16]. A linear trend model is also computed for sum of Values (actual & forecast) for the given year. Here, the sum squared error is 0.0513915, Mean squared error is 0.0042826, R squared values is 0.779349, standard error is 0.0654418 and the p-values (significance) is <0.0001. The forecast values with lower and upper confidence bound is shown in Table 2. Figure 5, shows the MATLAB results of the Nutrients, macrophytes and fish population. Wherein, when the nutrients are released into the lake there is steep increase and decrease in macrophytes and fish population. Phase plot shows the interaction between the parameters. Further, the stability nature of equilibrium points is shown in Figure 6. The results also shows that there is massive increase in macrophytes leaving no room for survival of fish population.

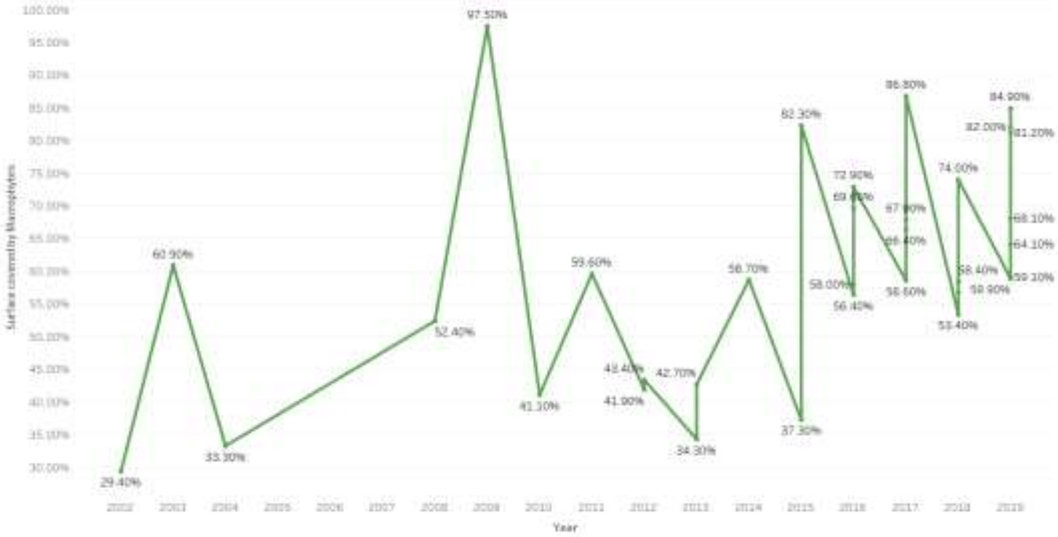

Figure 3: Macrophytes cover in Varthur lake from the year 2002-2019

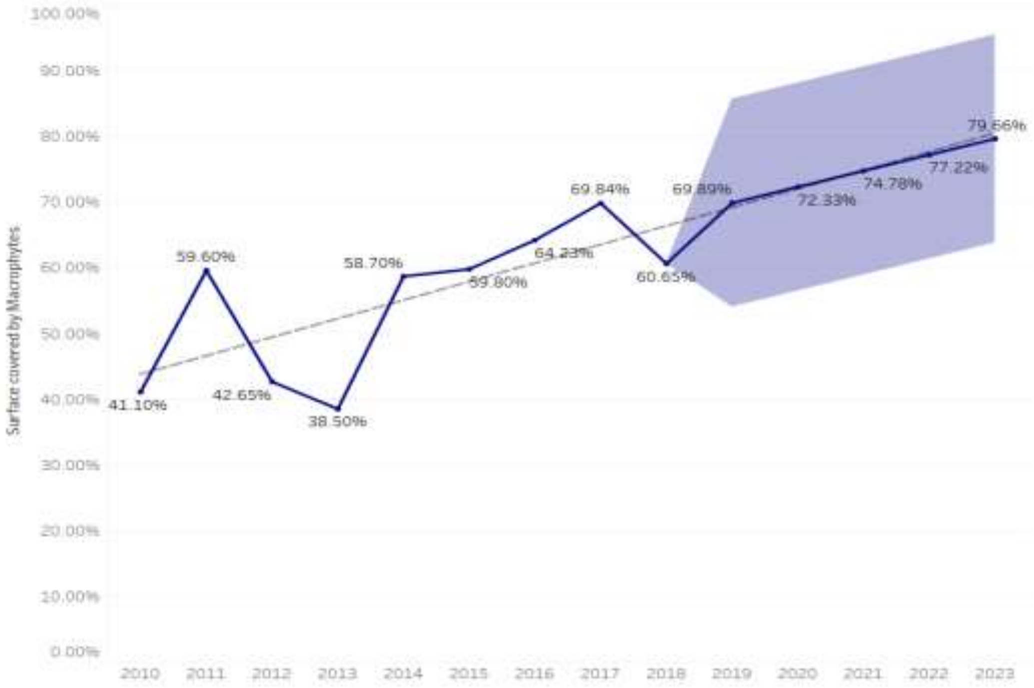

Figure 4: Average macrophytes cover forecast till 2023 with linear trend line.

Table 2: The Average values of macrophytes covering the surface of Varthur lake from 2010-2019

Timeline	Average Values
2010	41.10%
2011	59.60%
2012	42.65%
2013	38.50%
2014	58.70%
2015	59.80%
2016	64.23%
2017	69.84%
2018	60.65%
2019	71.19%

Timeline	Forecast values	Lower Confidence Bound	Upper Confidence Bound
2020	72.33%	56.85%	87.81%
2021	74.78%	59.30%	90.26%
2022	77.22%	61.74%	92.70%
2023	79.66%	64.18%	95.14%

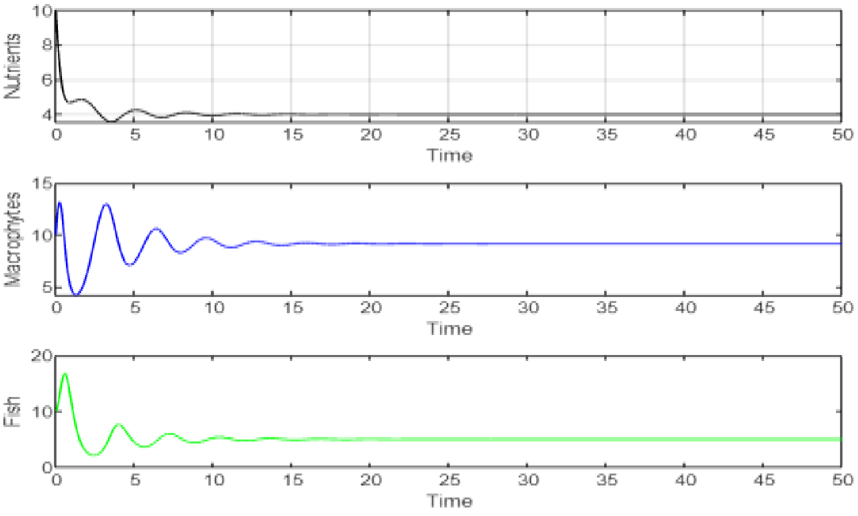

Figure 5: Variation of Nutrients, Macrophytes and Fish population with time.

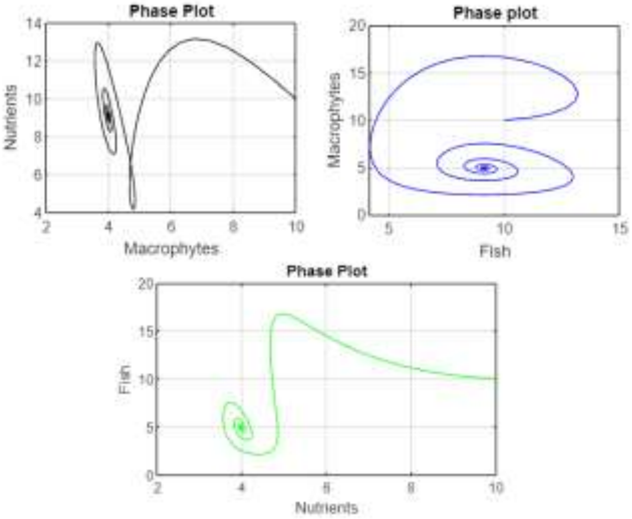

Figure 6: Phase plot showing interaction between Nutrients, Macrophytes and Fish with stability nature of equilibrium points.

CONCLUSION

A mathematical model is formulated to study the dynamics between Nutrients, macrophytes and its effects on fish population. The stability of the equilibrium points is studied and analysed with Routh-Hurwitz criteria. The equilibrium points and are conditionally stable whereas, the coexistence equilibrium point is stable. The study demonstrates the varthur lake cover analyses of macrophytes. Numerical simulation supports the analytical findings. The high coverage of macrophytes exhibits harmful effects on the survival of fish population, which is in turn triggers by the nutrients released into the varthur lake. The forecast also shows the increase in macrophytes by 2023 which will further degrade the lake. To protect the Varthur lake and similar lakes of Bangalore, the release of pollutants into the water body should be addressed immediately.

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