Original Article

Connectedness In Soft Topological Spaces

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Abstract

The main aim is to produce a new form of connectedness namely Soft Jc Connected. Further more we have introduced Soft Jc Seperated Sets and we have obtained some results using Soft Hyperconnected set. Further we have compared our Soft Jc Connected Space with Soft α Connected and Soft $\alpha\beta$ Connected spaces. Moreover we have proved Generalization of Intermediate Value Theorem with respect to Soft Jc Connected and we have discussed some properties of Soft Jc Connected Space in detail.

Keywords: Soft Jc-Closed, Soft Jc-Continuous, Soft Jc-Seperated, Soft Jc- Connected.

1.INTRODUCTION

The methodology of connectedness via various genera-lized open sets is not a new idea in topological spaces. Njastad [5] introduced the α-open sets and investigated the topological structure on the class of these sets; the α-open sets form a topology. The notion of Soft set theory was introduced in 1999 by Molodtsov [3]. Maji et al. [4] have initiated some operations on Soft sets. Further, many researchers paved way to the Soft set theory and its applications. 'Shabir and Naz [6] was initiated the notion of Soft topological spaces with a fixed set of parameters'**.** The authors of this paper have introduced a new class of Soft generalized closed set namely Soft Jc closed set[1] and studied their properties. In this paper, connectedness of a class of Soft Jc-open sets in a Soft topological space $\tilde{X}$ was introduced. The connectedness of this class on $\tilde{X}$, called Soft Jc-connectedness, turns out to be equivalent to connectedness of $\tilde{X}$ when $\tilde{X}$ is locally indiscrete or with finite Soft α-topology. An 'Intermediate-value-theorem' is obtained. The Soft hyperconnected spaces constitute a subclass of the class of Soft Jc-connected spaces. In this work Closed represents closed and Connected represents connected.

2.PRELIMINARIES: In this work, 'X refers to an initial universe', $P(X)$ is the power set of X,E,K denote the set of parameters and (X, τ_E), (Y, σ_K) denote Soft topological spaces where no Soft separation axioms are assumed unless it is explicitly stated.

Definition.2.1 A Soft Jc-Closed [1] if Sαcl(A_E) $\tilde{\subseteq}$ Int(U_E) whenever (A_E) $\tilde{\subseteq}$ (U_E) and (U_E) is Soft $\hat{g}$-open in ($\tilde{X}$,τ_E); the complement of Soft Jc Closed set is called a Soft Jc-open set.

Definition.2.2*:* A function f_c : ($\tilde{X}$,τ_E)→($\tilde{Y}$, σ_K) is called **Soft Jc-continuous** [2] if $f_c^{-1}(V_K)$ is Soft Jc-Closed in ($\tilde{X}$,τ_E) for every Soft Closed set (V_K) of ($\tilde{Y}$, σ_K)

Definition.2.3: [7] A Soft topological space ($\tilde{X}$,τ_E) is called Soft **Connected space** if $\tilde{X}$ cannot be written as a disjoint Soft union of two non-empty Soft open sets.
Definition.2.4. A Soft topological space ($\tilde{X}$,τ_E) is said to be
(1) '**locally-Soft-indiscrete** if all Soft open subset of ($\tilde{X}$,τ_E) is Soft Closed'.
(2) '**Soft hyperconnected** if all non-empty Soft open subset of ($\tilde{X}$,τ_E) is Soft dense in ($\tilde{X}$,τ_E)'
Theorem.2.5: A function f_c from $\tilde{X}$ to $\tilde{Y}$ is Soft Jc-continuous iff the inverse image of every Soft open set in $\tilde{Y}$ is Soft Jc-open in $\tilde{X}$.

3. SOFT Jc-CONNECTED SPACES

Definition.3.1. Any two Soft non-empty subsets A_E and B_E of a Soft topological-space $\tilde{X}$ are known to be "**Soft Jc-separated"** if $A_E \tilde{\cap}$ JcCl(B_E) = $\tilde{\emptyset}$ = JcCl(A_E) $\tilde{\cap}$ B_E. The well known fact is that two Soft Jc-separated-sets are dis-joint. If A_E & B_E are two S-ft Jc separated sets in $\tilde{X}$ with $\emptyset \neq C_E \tilde{\subseteq} A_E$ and $\emptyset \neq D_E \tilde{\subseteq} B_E$. Hence C_E & D_E will represent Soft Jc-separated-sets in $\tilde{X}$.
Definition.3.2. A Soft topological space $\tilde{X}$ with a Soft subset S is known to be Soft **Jc-Connected** in $\tilde{X}$ if 'S is not the union of two Soft Jc-separated-sets' in $\tilde{X}$.
Remark.3.3. A Soft infinite-space with cofinite Soft topology is Soft Jc-Connected.
Theorem.3.4. A Soft topological space $\tilde{X}$ is Soft Jc-Connected iff '$\tilde{X}$ cannot be expressed as the union of two dis-joint non-empty Soft Jc-open subsets of $\tilde{X}$'.
Proof. Let $\tilde{X}$ be Soft-Jc-Connected, and A_E and B_E be two dis-joint non-empty Soft Jc-open subsets of $\tilde{X}$ such that $\tilde{X}$ = $A_E \tilde{\cup} B_E$.
Then A_E and B_E are Soft Jc-closed in $\tilde{X}$.
Therefore, $A_E \tilde{\cap}$ JcCl(B_E) = $\emptyset$ = JcCl(A_E) $\tilde{\cap}$ B_E.
Then $\tilde{X}$ is not Soft Jc-Connected, Which contradicts our hypothesis.
This proves that '$\tilde{X}$ cannot be expressed as the union of two dis-joint non-empty Soft Jc-open subsets of $\tilde{X}$'.
Conversely, suppose that $\tilde{X}$ = $A_E \tilde{\cup} B_E$, $A_E \neq \emptyset \neq B_E$ and $A_E \tilde{\cap}$ JcCl(B_E) = $\emptyset$ = JcCl(A_E) $\tilde{\cap}$ B_E. Thus A_E and B_E be two disjoint non-empty Soft Jc-open subsets of $\tilde{X}$, contradiction occurs. Hence, $\tilde{X}$ is Soft-Jc-Connected.
Theorem.3.5. For a Soft topological space $\tilde{X}$, the below results are equi-valent:

1. $\tilde{X}$ is Soft Jc-Connected.
2. The only Soft sub-sets of $\tilde{X}$ that are both Soft Jc-open & Soft Jc-Closed in $\tilde{X}$ and the empty set.
3. There is no non-constant onto Soft Jc conti-nuous func-tion from '$\tilde{X}$ to a discrete Soft space that contains more than one-point'.

Proof.
(1)⇒(2). Proof is obvious by Theorem.3.4.
(2)⇒(3). Let $\tilde{Y}$ be a discrete-Soft-space, and f_c :($\tilde{X}$,τ_E)→($\tilde{Y}$, σ_K) be onto Soft Jc-continuous function.
Let $y_k \in \tilde{Y}$ and A_K = {y_k}.
Since f_c :($\tilde{X}$,τ_E)→ $\widetilde{(Y}$, σ_K) is Soft Jc-continuous and onto, by theorem 2.5, f^{-1} (A_E) is non-empty, Soft Jc-open and Soft Jc-Closed subset in $\tilde{X}$.
Since f^{-1} (A_E) is non-empty, f^{-1} (A_E) = $\tilde{X}$. That is, f_c is constant.
(3)⇒(1).Assume that $\tilde{X}$ is not Soft Jc-Connected. If $\tilde{X}$ = $A_E \tilde{\cup} B_E$, where A_E & B_E are non-empty subsets of $\tilde{X}$ such that JcCl(A_E) $\tilde{\cap}$ B_E = $\tilde{\emptyset}$ and JcCl(B_E) $\tilde{\cap}$ A_E = $\tilde{\emptyset}$.
Then A_E and B_E both are Soft Jc-open sets in $\tilde{X}$.
Assume that $\tilde{Y}$ = {$\tilde{\emptyset}$, $\tilde{X}$} with dis-crete Soft topo-logy.
We define a Soft map f_c :($\tilde{X}$,τ_E)→($\tilde{Y}$, σ_K) by $f_c(x)$ = $\tilde{\emptyset}$ if $x_e \tilde{\in} A_E$ and $f_c(x)$ = $\tilde{X}$ if x $\tilde{\in}$ B_E. Then f_c is non-constant Soft Jc-continuous and onto mapping, a contradiction to (3).
Lemma.3.6. If $\tilde{X}$ is Soft hyperconnected, therefore JcO($\tilde{X}$) $\tilde{\cap}$ JcCl($\tilde{X}$) = {$\tilde{\emptyset}$, $\tilde{X}$}.
Theorem.3.7. If a Soft space '$\tilde{X}$'is Soft hyperconnected, then $\tilde{X}$ is Soft Jc-Connected.
Proof. Obvious by Lemma 3.6 and Theorem 3.5.
Remark.3.8. A Soft topological space $\tilde{X}$ is Soft αβ-Connected iff $\tilde{X}$ is Soft Connected.
Theorem.3.9. If a Soft space $\tilde{X}$ is Soft Jc-Connected, then it is Soft Connected.
Proof. Suppose $\tilde{X}$ be Soft Jc-Connected. Then 'the only Soft subsets of $\tilde{X}$ that are both Soft Jc-open and Soft Jc-Closed in $\tilde{X}$' are $\tilde{\emptyset}$ and $\tilde{X}$.
Assume that the space $\tilde{X}$ is not Soft Connected, then there is a 'non-empty proper Soft subset A_E of $\tilde{X}$ that is both Soft open and Soft Closed'.

Which implies A_E is both Soft Jc-open and Soft Jc-Closed in $\tilde{X}$, a contradiction.

Example.3.10. $X = \{1,2,3\}$, $E = \{e_1, e_2\}$, $\tau = \{\tilde{\phi}, \tilde{X}, F_E\}$ where $F_{e1} = \{x_2\}$and $F_{e2} = \{\emptyset\}$ then ($\tilde{X}, \tau_E$) is Soft Connected but not Soft Jc Connected because $\tilde{X} = (e_1,x_2) \tilde{\cup} (e_2,x_1,x_2)$ where (e_1,x_2) and (e_2,x_1,x_2) are disjoint non-empty S-ft Jc open sets

Remark.3.11.

1. A Soft topological space $\tilde{X}$ is Soft Connected iff $\tilde{X}$ is Soft α-Connected.
2. A Soft topological space $\tilde{X}$ is Soft αβ-Connected iff $\tilde{X}$ is Soft α Connected.

Theorem 3.12. If a Soft space $\tilde{X}$ is Soft semi-Connected, then it is Soft Jc-Connected.

Proof. By this fact $JcO(\tilde{X}) \tilde{\subseteq} SO(\tilde{X})$ proof follows.

Remark.3.13.

1. If a Soft space $\tilde{X}$ is Soft T_1 or locally Soft in-discrete, then $JcO(\tilde{X}) = SO(\tilde{X})$.
2. A Soft T_1-space or locally Soft in-discrete space $\tilde{X}$ is Soft semi-Connected iff $\tilde{X}$ is Soft Jc-Connected.

Theorem.3.14. If a Soft space $\tilde{X}$ is locally Soft in-discrete, then $JcO(\tilde{X}) = SO(\tilde{X}). = \tau$.

Proof. It is sufficient to show that all Soft semi-open set is Soft open in $\tilde{X}$,when $\tilde{X}$ is locally Soft in-discrete.
Let A_E be a Soft s-open set in $\tilde{X}$.
Hence $A_E \tilde{\subseteq} Cl(Int(A_E))$. Since $\tilde{X}$ is locally Soft in-discrete, $Cl(Int(A_E)) = Int(A_E)$.

Theorem.3.15. "**Generalization of Intermediate value theorem**: Let $f_c : (\tilde{X}, \tau_E) \rightarrow \mathcal{R}$ be a Soft Jc-continuous-map from a Soft Jc-Connected-space $\tilde{X}$ to the real line $\mathcal{R}$. If x and y are two points of $\tilde{X}$ such that $a_e = f_c(x)$ and $b_e = f_c(y)$, then every real number r between a_e and b_e is attained at a point in $\tilde{X}$".

Proof. Assume that there is no Soft point $c_e \tilde{\in} \tilde{X}$, st $f_c(c_e) = r$.
Thus $A_E = (-\infty, r)$ & (r, ∞) are distinct Soft open sets in $\mathcal{R}$. 'Since f_c is Soft Jc continuous', $f_c^{-1}(A_E)$ and $f_c^{-1}(B_E)$ are distinct Soft Jc-open sets in $\tilde{X}$ & $\tilde{X} = f_c^{-1}(A_E) \tilde{\cup} f_c^{-1}(B_E)$,contradicts.

4.PROPERTIES OF SOFT Jc CONNECTED SPACES

Theorem.4.1. If A_E be a Soft Jc-Connected set of a Soft topological space $\tilde{X}$ and U_E, V_E are 'Soft Jc-separated sets' of $\tilde{X}$ such that $A_E \tilde{\subseteq} U_E \tilde{\cup} V_E$, then either $A_E \tilde{\subseteq} U_E$ or $A_E \tilde{\subseteq} V_E$.

Proof. "Since $A_E = (A_E \tilde{\cap} U_E) \tilde{\cup} (A_E \tilde{\cap} V_E)$",
we have $(A_E \tilde{\cap} U_E) \tilde{\cap} JcCl(A_E \tilde{\cap} V_E) \tilde{\subseteq} U_E \tilde{\cup} JcCl(V_E) = \tilde{\emptyset}$.
If $A_E \tilde{\cap} U_E$ and $A_E \tilde{\cap} V_E$ are non-empty, then A_E is not Soft Jc-Connected, a contradiction.
Therefore, either $A_E \tilde{\subseteq} U_E$ or $A_E \tilde{\subseteq} V_E$.

Theorem.4.2. If A_E be a Soft Jc-Connected set of a Soft topological space $\tilde{X}$ and
$A_E \tilde{\subseteq} N_E \tilde{\subseteq} JcCl(A_E)$, then N_E is Soft Jc-Connected.

Proof. Our assumption is that N_E is not Soft Jc-Connected set. ∃ Soft Jc-separated sets U_E and V_E such that $N_E = U_E \tilde{\cup} V_E$.
Then by the above theorem, either $A_E \tilde{\subseteq} U_E$ or $A_E \tilde{\subseteq} V_E$. If $A_E \tilde{\subseteq} U_E$, then $JcCl(A_E) \tilde{\cap} V_E = \emptyset$, a contradiction. Proof follows

Corollary.4.3. If A_E is a Soft Jc-Connected subset of a Soft topological space $\tilde{X}$, then $JcCl(A_E)$ is Soft Jc-Connected.

Theorem.4.4. Suppose A_E and B_E be Soft subsets of a Soft topological space $\tilde{X}$. If A_E & B_E are Soft Jc-Connected and not Soft Jc-separated, then $A_E \tilde{\cup} B_E$ is Soft Jc-Connected.

Proof. Assume that $A_E \tilde{\cup} B_E$ is not Soft Jc-Connected. Therefore there exist Soft Jc-separated sets C_E and D_E in $\tilde{X}$ such that $A_E \tilde{\cup} B_E = C_E \tilde{\cup} D_E$.
By Theorem 4.1, either $A_E \tilde{\subseteq} C_E$ or $A_E \tilde{\subseteq} D_E$ and $B_E \tilde{\subseteq} C_E$ or $B_E \tilde{\subseteq} D_E$.
If $A_E \tilde{\subseteq} C_E$ and $B_E \tilde{\subseteq} C_E$, then $(A_E \tilde{\cup} B_E) \tilde{\subseteq} C_E$ and $D_E = \tilde{\emptyset}$, a contradiction.
If $A_E \tilde{\subseteq} C_E$ and $B_E \tilde{\subseteq} D_E$, then A_E and B_E are Soft Jc-separated sets in $\tilde{X}$, a contradiction.

Theorem.4.5. let $\{B\gamma;\ \gamma \tilde{\in} \Gamma\}$ is a non-empty family of Soft Jc-Connected-subsets of a Soft topological space $\tilde{X}$ such that $\tilde{\cap} B\gamma \neq \tilde{\emptyset}$, then $\tilde{\cup} B\gamma$ is Soft Jc-Connected.

Proof. Suppose that $N = \tilde{\cup} B\gamma$ and N is not Soft Jc-Connected.
Then $N = U_E \tilde{\cup} V_E$, where U_E and V_E are Soft Jc-separated sets in X. Since $\tilde{\cap} B\gamma \neq \tilde{\emptyset}$, there is a Soft point x_e in $\tilde{\cap} B\gamma$.
Since $x_e \tilde{\in} N$, either $x_e \tilde{\in} U_E$ or $x_e \tilde{\in} V_E$.
assume that $x_e \tilde{\in} U_E$. Since $x_e \tilde{\in} \tilde{\cap} B\gamma$, $B\gamma$ and U_E intersects for every γ.
'By Theorem.4.1', $B\gamma$ should be in U_E for each $\gamma \tilde{\in} \Gamma$. Then $N \tilde{\subseteq} U_E$, a contradiction.
The proof now follows.

Theorem.4.6. Suppose $\{An;\ n \tilde{\in} \tilde{N}\}$ is a Soft infinite-sequence of Soft Jc-Connected subsets of a Soft topological space $\tilde{X}$ & An $\tilde{\cap}$ An+1$\neq$ $\tilde{\emptyset}$ for all n $\tilde{\in}$ N, then $\tilde{\cup}$An is Soft Jc-Connected.

Proof. By induction hypothesis and by Theorem.4.5 the proof is obvious.

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