Original Article

ALEPH FUNCTION WITH TRANSFORMATION OF DOUBLE INTEGRAL

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Abstract

In this paper, we studied double integral transform of Aleph ( ) ℵ -function which leads to formation of another important process of augmenting parameters in Aleph ( ) ℵ - function. Aleph function of executives is central factor for any intrigue's development and adequacy Here we saw some of general character in the form of result and on specializing parameters suitably, yields several interesting results as special cases.

KEYWORD - Aleph function, parameters**,** function general character

INTRODUCTION

Aleph function in the field of mathematicians is consider as a tool which gives to development qualities. Showing the influence and obliging workplace is highly obligation of the pioneer. Regardless, whenever it is said that reorganization bearing those specialists are plans to obtain required targets would see making unsatisfied workforce. Abnormal structure with two social functions estimations is seen. In this way starting late referenced styles, explanation for interactions of road affiliation style should be approved which would help in exchanging agents about as ampleness and aleph function measures. It is seen that invigorated learning and numeric values statistics applications as related considerations. Administrator own one of lives are surprisingly affected by both verified information and numeric values statistics applications generally. As per our requirements be on gotten handle on was picked up to gut how virtuoso's ground statistics applications and animated appreciation are affected by various parts, for instance, aleph function, conjugal status, heading and experience. In like way to close gigantic relationship was insisted between position statistics applications and engaged insight and it got affected to dependably key degree by work obligation and wedding status. Many trials were taken to take a gander at impact of digits on Aleph function of and execution of association. Whole plan perfect situation of supervisor digits is higher bit of leeway. In that limiting value of current time it is titanic for relationship to give most astonishing digits as it gives result into standing headway of association. The survey of Aleph function is stated as test pro viewpoints regarding interchange characters of their affiliations. Total nine area of instruments where every point joined. Paper shows that there are fairly couple of undeniable evaluations paying special mind to estimation invariance issues concerning numeric values related mindsets over different work settings.

In current paper, we found finite integrals with product of generalized polynomial sets and multivariable Aleph (ℵ)-function. These integrals are identical in nature as key formula from which we can evaluate its special conditions as integrals involving large number of simpler special functions and polynomials

Integral function of aleph function

ℵ function is defined as

$$\aleph[x] = \aleph^{m,n}_{p,q,x;r}\ [x|\begin{cases}(aj, Aj)[\tau(aij, Aji)]n+1 \\ (bj, Bj)[\tau(bij, Bij)]n+1\end{cases}|\quad]$$

$$\frac{1}{2\pi i}\int_L \theta(S)ds$$

Where i = $\sqrt{1}$

$\theta(S)ds= \frac{\prod_{j=1}^{m}\tau(b_{j-B_{JS}})\prod_{I=1}^{n}(1-a_{i+A})}{\sum_{i=1}^{r}\tau_i\{\prod_{j=1}^{p}\tau(1-b_{j-B_{JS}})\prod_{j=1+n}^{n}(-a_{i+A})}$

Where p, q, m, n are different types of integers which fulfilling the conditions of
$0 \le m \le p$, $0 \le n \le q$, A_i, B_j,, A_{ji}, B_{ij} are real and a_i, b_j, a_{ij}, b_{ij} are numbers which are complex
Which gives that
$A_j\,(b_h + v) \neq B_h\,(a_i\text{-}v\text{-}k)$
Where k and v are real numbers

Some notations

$A^{*} = (a_i, A_i)_{1,n}, [\tau_i(a_{ij}, A_{ij})]_{n+1,p}$; $B^{*} = (b_j, B_j)_{1,m}, [j(b_{ij}, BB_{i,j})]_{m+1,q}$;
$A^{**} = (c_i, C_i)_{1,r}, [\tau_i(c_{ij}, C_{ij})]_{r+1,m}$; $B^{**} = (d_j, D_j)_{1,s}, [j(d_{ij}, BD_{i,j})]_{S+1,q}$;
If we use $\tau = 1,2,3,\ldots.$ And $r = 1,2,3,\ldots.$ In above equtation then $\aleph - can\ converts\ into\ H - function\ of\ fox's$

Double integral transformation of $\aleph - function$

$\iint_0^{\infty} x^{a-1}.y^{b-1}(x+y)^{\mu}\, \aleph_{u,v,x,r}^{f,g}[\gamma\,(x+y)|\,{}_{D}^{C}]$
$\aleph_{p,q,r,x}^{n,m}\,[tx^k y^s (y+x)^{r}|_{B}^{A}]$ dx d y =

$(2\pi)^{(1-d)\,(g+f-1\backslash 2\mu--1\backslash 2\, v)+1\backslash 2}\ DD^{\sum_i^u d - \sum_j^v c + (A-1\backslash 2)(v-u)}$

$$\frac{s^{a-1\backslash 2} k k^{b-1}}{\lambda^{a-b-\mu}(s+k)^{\ a+b-1\backslash 2}}\aleph_{\rho+p+Dv,q+p+Du}^{n+Dg,m+\rho+Df}$$

$$\left[\frac{t\delta D^{D(u-v)}}{\gamma^D}\Big|\begin{matrix}\triangle((D,1-A-D)),\triangle(s,1-a),\triangle(k,1-b),AA^{*},\triangle(D,1-A-d)\ldots\triangle(D,1-A-d)\\ \triangle((D,1-A-C)),B^{*}\triangle(s+k,1-a-b),\triangle(D,1-A-c),AA^{*},\triangle(D,1-A-d)\ldots\triangle(D,1-A-d)\end{matrix}\right]$$

$$\lambda = \frac{S^k K^s}{s+k^{k+s}}\ ,\rho = k+s, D = s+k+r\ , A = \alpha+\beta+\mu$$

$0 \le D_g \le D_U \le D_v \le D_u + q - p$, $u + v - 2g \le 2f \le 2v$,$0 \le p \le n$,
$P + q - 2n < 2m \le 2q$
Re (max $\frac{d_j}{\delta_j} + D$ min $\frac{b_{ij}}{\beta_{ij}}) > Re(-A) >$ Re ([D $\left(\frac{S-\alpha}{S}, \frac{K-\beta}{K}, \alpha_i\right) + C_t - D - 1]$
I = 1,2,3,…..f; j = 1,2,…n ; t = 1,2,3….,g ; u,
Re (max $\frac{d_j}{\delta_j} + A) - uD + v + \frac{1}{2}$ D (Du-Dv+1) > D (Du-Dv)
Re max ([D $\left(\frac{S-\alpha}{S}, \frac{K-\beta}{K}, \alpha_i\right)$
I = 1,2,….u; j=1,2….v; l=1,2….m |arg $\gamma| \le (f + g - \frac{1}{2}$ v—$\frac{1}{2}$ u) π
|arg |≤ (m+n-$\frac{1}{2}\ q - \frac{1}{2}\ p)\pi$, Re ($\alpha + s\frac{b_j}{\beta_j}$) > Re ($\beta + k\frac{b_j}{\beta_j}) > 0\ , j = 1,2 \ldots m$
In this way double integral converges

Main Integral

$\iint_0^1 \frac{(1-p)}{(1-pq)}$ u q $\frac{(1-q)}{(1-pq)}\ \frac{(1-pq)}{(1-p)(1-q)})$
$A_m^l\ \frac{(1-p)}{(1-pq)}$ u q $A_{m'}^{l'}\ \frac{(1-p)}{(1-pq)} uq\quad \aleph_{P,Q,X,Y}^{L,M}\ \frac{(1-q)}{(1-pq)}\quad dxdy$

$\sum_{h=0}^{m/l}\frac{(-l)_{l,h)}}{h!}\ B_{l,h}\ \frac{(1-p)}{(1-pq)} uq\ \sum_{h=0}^{m\prime/l\prime}\frac{(-l\prime)_{l\prime,h\prime)}}{h!}\ C_{l,h}\ \frac{(1-p)}{(1-pq)} uq$ Υ (h+2h $_1$ +δ)

$\aleph_{P+1,y+1,\gamma_1;x}^{L,M+1}$ (1-α;1),$(c_i C_i)$ $_{1,M}(\gamma_j(c_{ij}, C_{ij}))$ $_{M+1,}P_i; x$ |u
$(d_i D_i)$ $_{1,l}(\gamma_j(d_{ij}, D_{ij}))$ $_{l+1,q_{1,x}}\gamma_i$(1-h-2$h_1 - \beta - \alpha$)|u

Given by Re(β+α+cj/αj]>0 ,[arg u]<$\frac{\tau.\pi}{2}$
L and l' are positive integer value and coefficient B' $_{m\,h}$(l,h>0) and A' $_{l\,h}$ (l,h_1≥0)which is seen to be constant ,complex , real having finite integral
Proof –
We can take the following expression
$A_m^l\ \frac{(1-p)}{(1-pq)}$ u q $A_{m'}^{l'}\ \frac{(1-p)}{(1-pq)} uq\quad \aleph_{P,Q,X,Y}^{L,M}\ \frac{(1-q)}{(1-pq)}\ u$

$\sum_{h=0}^{m/l} \frac{(-l)_{l,h)}}{h!} B_{l,h} \frac{(1-p)}{(1-pq)} uq \sum_{h=0}^{m\prime/l\prime} \frac{(-l\prime)_{l\prime,h\prime)}}{h!} C_{l,h} \frac{(1-p)}{(1-pq)} uq$

$\frac{1}{2\pi\vartheta} \int k \frac{\prod_{i=1}^{l} \gamma(c_i+C_i)\prod_{i=1}^{m}\gamma(1-b_i-B_i s)}{\sum_{j=1}^{r} \delta \prod_{i=L+1}^{p_i}\gamma(b_{ij}+B_{ij} s)\prod_{i=L+1}^{p_i}\gamma(1-c_{ij}-C'_{ij}S)} \quad \frac{(1-p)}{(1-pq)} uq$

Multiply both side by following

$\frac{(1-p)}{(1-pq)}$ q $\frac{(1-q)}{(1-pq)}$ $\frac{(1-pq)}{(1-p)(1-q)}$

And then integrate we will get the following result

$\frac{(1-p)}{(1-pq)}$ q $\frac{(1-q)}{(1-pq)}$ $\frac{(1-pq)}{(1-p)(1-q)}$ $A_m^l \frac{(1-p)}{(1-pq)}$ u q

$A_{m\prime}^{l\prime} \frac{(1-p)}{(1-pq)} uq \quad \aleph_{P,Q,X,Y}^{L,M} \frac{(1-q)}{(1-pq)} u$

$= \sum_{h=0}^{m/l} \frac{(-l)_{l,h)}}{h!} B_{l,h} \frac{(1-p)}{(1-pq)} uq \sum_{h=0}^{m\prime/l\prime} \frac{(-l\prime)_{l\prime,h\prime)}}{h!} C_{l\prime,h\prime} \frac{(1-p)}{(1-pq)} uq$

$\frac{(1-p)}{(1-pq)}$ u q $\frac{(1-p)q}{(1-pq)}$ $\frac{(1-q)}{(1-pq)}$ $\frac{(1-pq)}{(1-p)(1-q)}$

$\frac{1}{2\pi\vartheta} \int k \frac{\prod_{i=1}^{l} \gamma(c_i+C_i)\prod_{i=1}^{m}\gamma(1-b_i-B_i s)}{\sum_{j=1}^{r} \delta \prod_{i=L+1}^{p_i}\gamma(b_{ij}+B_{ij} s)\prod_{i=L+1}^{p_i}\gamma(1-c_{ij}-C'_{ij}S)} \frac{(1-q)}{(1-pq)} u$ ds

If we integrate left side with respect to p and q within 0 and 1 we get result

$\iint_0^\infty \varphi(e+f)e^{\beta-1} f^{\alpha-1} A_m^l[e] A_{m\prime}^{l\prime}[e^2] \aleph_{q_j\delta_j\ r}^{L,M}[f]\ de\ df$

$\sum_{h=0}^{m/l} \frac{(-l)_{l,h)}}{h!} B_{l,h} \frac{(1-p)}{(1-pq)} uq \sum_{h=0}^{m\prime/l\prime} \frac{(-l\prime)_{l\prime,h\prime)}}{h!} C_{l,h} \frac{(1-p)}{(1-pq)} uq$ ϒ (h+βh₁ +β) $\int_0^\infty \varphi(\rho)\rho^{\beta+\alpha+h+2h_1-1}$

$\aleph_{P+1,y+1,\gamma_1;x}^{L,M+1}$ (α₁;1),$(c_i C_i)$ $_{1,M}(\gamma_j(c_{ij,}C_{ij}))$ $_{M+1,};x\ |\rho$

$(d_i D_i)$ $_{1,l}(\gamma_j(d_{ij,}D_{ij}))$ $_{l+1,q_{1,x}}\gamma_i$(1-h-2$h_1-\beta)|\rho$

Given that Re (β + α + c_i /α_i)> 0 Land L' are arbitrary positive integer and coefficients C' $_{m,h}$, D' $_{m'h'}$ (m,h ≥0 ,m' ,h_1≥ 0) are arbitrary constant ,real or complex

As we have

$A_m^l[e] A_{m\prime}^{l\prime}[e^2] \mu_{q_j\delta_j,\rho,r}^{L,M}$(f)

$\sum_{h=0}^{m/l} \frac{(-l)_{l,h)}}{h!} B_{l,h}[e]^h \sum_{h=0}^{m\prime/l\prime} \frac{(-l\prime)_{l\prime,h\prime)}}{h!} C_{l,h}\ [e^2]^h$

$\frac{1}{2\pi\vartheta} \int k \frac{\prod_{i=1}^{l} \gamma(c_i+C_i)\prod_{i=1}^{m}\gamma(1-b_i-B_i s)}{\sum_{j=1}^{r} \delta \prod_{i=L+1}^{p_i}\gamma(b_{ij}+B_{ij} s)\prod_{i=L+1}^{p_i}\gamma(1-c_{ij}-C'_{ij}S)}$ ds

Now multiplying both side by $\varphi(e+f)e^{\beta-1}f^{\alpha-1}$ and integrating with respect to e and f from both side between 0 to ∞ we get our result

Some special cases

By applying the result in the case of hermit's polynomial and by putting the value l=l'=2 ,

C '$_{m,h}$ = $(-1)^h$

D' $m_1 h_1$ = $(-1)^{h1}$

$A_m^2(p) \to p^{m/2} H_m \frac{1}{2\sqrt{p}}$ and

$$A_m^2\, p^2 \to p^m\, H_m \frac{1}{2x}$$

We get the following consequence's

$$\iint_0^1 \frac{(1-p)}{(1-pq)} q \frac{(1-q)}{(1-pq)} \frac{(1-pq)}{(1-p)(1-q)} \frac{(1-p)}{(1-pq)}$$

$\frac{(1-p)}{(1-pq)} uq\ H_m \frac{1}{2\sqrt{\frac{(1-p)}{(1-pq)}} uq}$

$H_m \frac{1}{2\frac{(1-p)}{(1-pq)} uq} \quad \aleph_{P+1,y+1,\gamma_1;x}^{L,M} \frac{(1-p)}{(1-pq)}\ dx\ dy$

$\sum_{h=0}^{m/2} \frac{(-m)_{2h)}}{h!} (-1)^h u^h \sum_{h=0}^{m/2} \frac{(-m)_{2h)}}{h!} (-1)^h (u\ 2)^h)$ (h+2$h_1 + \beta$)|

$\aleph_{P+1,q+1,\gamma_1;x}^{L,M}$ (1-α;1),$(c_i C_i)$ $_{1,M}\left(\gamma_j(c_{ij,}C_{ij})\right)$ $_{M+1,}; x$ |
($d_i D_i$) $_{1,l}\left(\gamma_j(d_{ij,}D_{ij})\right)$ $_{l+1,q_{1,x}}\gamma_i$(1-h-2$h_1 - \beta - \alpha$)

Valid under similar conditions
Let us take

$$\iint_0^{\rho} \varphi(e+f)e^{\beta+\frac{m}{2}+n-1} f^{\alpha-1} H_m \frac{1}{\sqrt[2]{p}} H_m \frac{1}{2p} \aleph_{P,q,\gamma_1;x}^{L,M} [u]\, dp\, dq$$

$$\sum_{h=0}^{m/2} \frac{(-m)_{2h}}{h!} (-1)^h \sum_{h=0}^{m/2} \frac{(-m)_{2h}}{h!} (-1)^h \int_0^{\rho} \varphi(\times) x^{\beta+\alpha+h-2h_1\ -1} \gamma(h+2h+\beta)$$

$\aleph_{P+1,y+1,\gamma_1;x}^{L,M+1}$ (1-α;1),$(c_i C_i)$ $_{1,M}\left(\gamma_j(c_{ij,}C_{ij})\right)$ $_{M+1,}P_i; x$ |u dx
($d_i D_i$) $_{1,l}\left(\gamma_j(d_{ij,}D_{ij})\right)$ $_{l+1,q_{1,x}}\gamma_i$(1-h-2$h_1 - \beta - \alpha$)|u

Suited under the stated condition

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