Original Article

Boundary Value Problem For A Degenerate Equation Of Odd Order With Variable Coefficients

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1. Introduction and problem statement.

In the region $\Omega=\{(x,y):0<x,y<1\}$ for the equation

$$(-1)^n D_x^{2n+1}u(x,y)+K(y)l(u(x,y))=0, \qquad (1)$$

where $D_z^i=\dfrac{\partial^i}{\partial z^i}, n\in N,$

$$l(u(x,y))=(-1)^s\frac{\partial^{2s}u(x,y)}{\partial y^{2s}}+\frac{\partial^{s-1}}{\partial y^{s-1}}\left((-1)^{s-1}p_{s-1}(y)\frac{\partial^{s-1}u(x,y)}{\partial y^{s-1}}\right)+\dots$$

$$+\frac{\partial}{\partial y}\left(-p_1(y)\frac{\partial u(x,y)}{\partial y}\right)+p_0(y)u(x,y),$$

$$0\le p_j(y)\in C^j([0,1]),\ j=0,1,\dots,s-1,s\in N.$$

$$K(y)\in C[0,1],K(y)>0,y\in(0,1],K^{(j)}(y)=O\left(y^{m-j}\right),y\to+0,0\le m<s,j\in N\cup\{0\},$$

Let's study the next problem.

Problem A. Find a regular solution of equation (1) from class $u(x,y)\in C_{x,y}^{2n+1,2s}(\Omega)\cap C_{x,y}^{2n,2s-1}(\overline{\Omega})$, that satisfies the boundary conditions

$$D_y^k u(x,0)=D_y^k u(x,1)=0,k=\overline{0,s-1}, \qquad (2)$$

$$D_x^j u(0,y)=\varphi_j(y),j=\overline{0,n}, \qquad (3)$$

$$D_x^r u(1,y)=\varphi_{n+1+r}(y),r=\overline{0,n-1}, \qquad (4)$$

$$\int_0^1\left(l(u(x,y))\right)^2dy<\infty. \qquad (5)$$

In the case of $K(y)=1$, the model equation (1) was studied in [1], where a technique for constructing a fundamental solution was developed. Using this technique, for the equations

$$D_x^{2n}u-\left(-1\right)^n D_y^1 u=0,\ D_x^{2n+1}u+\left(-1\right)^n D_y^2 u=0,\ D_x^{2n+1}u+\left(-1\right)^n D_y^1 u=0,$$

in papers [2-4], respectively, fundamental solutions in the form of improper integrals are found and a theory of potential is constructed.

When studying the so-called stationary viscous transonic linear equation (or BT-equation)

$$D_x^3 u\left(x,y\right)+D_y^2 u\left(x,y\right)+\frac{a}{y}D_y^1 u\left(x,y\right)=f\left(x,y\right),$$

in the cases a=0, using the similarity method and self-similar solution, the properties of fundamental solutions expressed in terms of special functions were constructed and studied in [5–7], and some boundary value problems were also solved. In addition, in [8-9], the Green's functions of some external boundary value problems in the cases a=0 and a=1 are explicitly constructed. The case of arbitrary a was studied in [10]. In this article, for equation (1), which is a certain generalization of the BT equation, problem A is studied.

2. Spectral problem.

Considering condition (2), consider the following eigenvalue problem:

$$\begin{cases} l\left(Y\left(y\right)\right)=\lambda\dfrac{Y\left(y\right)}{K\left(y\right)}, \\ Y^{(k)}\left(0\right)=Y^{(k)}\left(1\right)=0, k=0,1,...,s-1. \end{cases} \tag{6}$$

Let us prove that if there are eigenvalues, then they are all positive. Indeed, we have

$$\int_0^1 Y\left(y\right)l\left(Y\left(y\right)\right)dy=\lambda\int_0^1\frac{Y^2\left(y\right)}{K\left(y\right)}dy,$$

from here

$$\int_0^1\left\{\left(Y^{(s)}\left(y\right)\right)^2+p_{s-1}\left(y\right)\left(Y^{(s-1)}\left(y\right)\right)^2+...+p_0\left(y\right)Y^2\left(y\right)\right\}dy=\lambda\int_0^1\frac{Y^2\left(y\right)}{K\left(y\right)}dy,$$

because $Y^{(s)}\left(y\right)\neq 0, Y\left(y\right)\neq 0,$ then $\lambda>0.$

Let us show that problem (6) has eigenvalues. Because $\lambda\neq 0,$ then there is a Green's function $G\left(x,\xi\right)$ of problem (6), and $G\left(x,\xi\right)=G\left(\xi,x\right),$ hence problem (6) can be reduced to an equivalent integral equation of the following form:

$$Y\left(y\right)=\lambda\int_0^1\frac{1}{K\left(\xi\right)}G\left(y,\xi\right)Y\left(\xi\right)d\xi. \tag{7}$$

Note that $G\left(y,\xi\right)=O\left(y^{s-1+\varepsilon}\right), y\to+0, \varepsilon>0.$ From (7) we obtain an integral equation with a symmetric kernel

$$\frac{Y\left(y\right)}{\sqrt{K\left(y\right)}}=\lambda\int_0^1\frac{G\left(y,\xi\right)}{\sqrt{K\left(y\right)K\left(\xi\right)}}\frac{Y\left(\xi\right)}{\sqrt{K\left(\xi\right)}}d\xi, \tag{8}$$

we introduce the notation

$$Y\left(y\right)=\frac{Y\left(y\right)}{\sqrt{K\left(y\right)}}, G\left(y,\xi\right)=\frac{G\left(y,\xi\right)}{\sqrt{K\left(y\right)K\left(\xi\right)}},$$

$$\int_0^1 G\left(y,\xi\right)dy<\infty, \int_0^1\int_0^1 G\left(y,\xi\right)dyd\xi<\infty,$$

$$G(y,\xi)=\sum_{k=1}^{\infty}\frac{Y(y)Y(\xi)}{\lambda_k}.$$

then from (8) we obtain an integral equation with a symmetric kernel

$$Y(y)=\lambda\int_0^1 G(y,\xi)Y(\xi)\,d\xi.$$

Hence problem (6) has at most a countable number of eigenvalues. Arrange the eigenvalues in ascending order:

$$\lambda_1\le\lambda_2\le\ldots\lambda_n\le\ldots$$

In what follows, we will assume that

$$\int_0^1\frac{Y_k^2(y)}{K(y)}dy=1, k=1,2,\ldots.$$

hence, taking into account Bessel's inequality, we have

$$\sum_{k=1}^{\infty}\frac{Y_k^2(y)}{\lambda_k^2}\le\int_0^1\frac{G^2(y,\xi)}{K(\xi)}d\xi<\infty. \tag{9}$$

Let us find the conditions under which the given function is expanded into a series in terms of eigenfunctions $Y_k(y), k=1,2,\ldots.$. The theorem is true.

Theorem 1. Let the function satisfy the conditions**:**

1. $\dfrac{\varphi(y)}{\sqrt{K(y)}}\in C[0;1]$;
2. $\varphi(y)\in C^{2s}[0;1]$;
3. $\varphi^{(i)}(0)=\varphi^{(i)}(1)=0, i=0,1,\ldots,s-1,$

then it can be expanded into a uniformly convergent series of the form

$$\varphi(y)=\sum_{k=1}^{\infty}\varphi_k Y_k(y),$$

where

$$\varphi_k=\int_0^1\frac{\varphi(y)Y_k(y)}{K(y)}dy.$$

Proof. We have equality

$$\varphi(y)=\int_0^1 G(y,\xi)l(\varphi(\xi))\,d\xi,$$

from here

$$\frac{\varphi(y)}{\sqrt{K(y)}}=\int_0^1\frac{G(y,\xi)}{\sqrt{K(y)K(\xi)}}\sqrt{K(\xi)}l(\varphi(\xi))\,d\xi,$$

or

$$\frac{\varphi(y)}{\sqrt{K(y)}}=\int_0^1 G(y,\xi)\sqrt{K(\xi)}l(\varphi(\xi))\,d\xi.$$

Using the Hilbert-Schmidt theorem, we get

$$\frac{\varphi(y)}{\sqrt{K(y)}}=\sum_{k=1}^{\infty}\varphi_k Y_k(y), \tag{10}$$

where

$$\varphi_k=\int_0^1\frac{\varphi(\xi)}{\sqrt{K(\xi)}}Y_k(\xi)d\xi=\int_0^1\frac{\varphi(\xi)Y_k(\xi)}{K(\xi)}d\xi,$$

from (10) we have

$$\varphi(y)=\sum_{k=1}^{\infty}\varphi_k Y_k(y).$$

Theorem 1 is proved.

3. Existence of a solution. We assume that the boundary functions $\varphi_i(y), i=\overline{0,2n},$ satisfy the conditions of Theorem 1. Then

$$\varphi_i(y)=\sum_{j=0}^{+\infty}\varphi_{i,j}Y_j(y),$$

where

$$\varphi_{i,j}(y)=\int_0^1\varphi_i(y)\frac{Y_j(y)}{K(y)}dy.$$

With respect to the variable x, we obtain the problem (for convenience, we will assume that the eigenvalues have the form $\lambda_j^{2n+1}, j=1,2,...$)

$$\begin{cases} X_j^{(2n+1)}(x)+(-1)^n\lambda_j^{2n+1}X_j(x)=0, \\ X_j^{(i)}(0)=\varphi_{i,j}, i=\overline{0,n}, \\ X_j^{(s)}(1)=\varphi_{i+n+1,j}, s=\overline{0,n-1}. \end{cases} \tag{11}$$

The characteristic equation for (11) has the form

$$\mu_j^{2n+1}=(-1)^{n+1}\lambda_j^{2n+1}. \tag{12}$$

Consider 2 cases:

1. Let $n=2s+1,$ then

$$\mu_{j,p}=\lambda_j(\alpha_p+i\beta_p),$$

where

$$\alpha_p=\cos\theta_p, \beta_p=\sin\theta_p, \theta_p=\frac{2\pi p}{4s+3}, p=\overline{0,(4s+2)},$$

$\alpha_p>0$ for $p=\overline{0,s};\overline{3s+3,4s+2}, \alpha_p<0$ for $p=\overline{s+1,3s+2}.$

The general solution of equation (11) has the form

$$X_j(x)=a_{1,j}e^{\lambda_j x}+\sum_{p=1}^{s}e^{\lambda_j\alpha_p x}\left(b_{p,j}\cos\lambda_j\beta_p x+b_{p+s,j}\sin\lambda_j\beta_p x\right)+$$

$$+\sum_{p=s+1}^{2s+1}e^{\lambda_j\alpha_p x}\left(b_{p,j}\cos\lambda_j\beta_p x+b_{p+s+1,j}\sin\lambda_j\beta_p x\right).$$

Note that the first row α_p is positive, while the second row is negative. We have

$$D_x^i X_j\left(x\right)=\lambda_j^i\left(a_{1,j}e^{\lambda_j x}+\sum_{p=1}^{s}e^{\lambda_j\alpha_p x}\left(b_{p,j}\cos\left(\lambda_j\beta_p x+i\theta_p\right)+b_{p+s,j}\sin\left(\lambda_j\beta_p x+i\theta_p\right)\right)+\right.$$

$$\left.+\sum_{p=s+1}^{2s+1}e^{\lambda_j\alpha_p x}\left(b_{p,j}\cos\left(\lambda_j\beta_p x+i\theta_p\right)+b_{p+s+1,j}\sin\left(\lambda_j\beta_p x+i\theta_p\right)\right)\right),i=\overline{0,\left(2s+1\right)}.$$

To determine the unknowns $a_{ij,p}$, we obtain a system of equations

$$\begin{cases} a_{1,j}+\sum\limits_{p=1}^{s}\left(b_{p,j}\cos i\theta_p+b_{p+s,j}\sin i\theta_p\right)+ \\ +\sum\limits_{p=s+1}^{2s+1}\left(c_{p,j}\cos i\theta_p+c_{p+s+1,j}\sin i\theta_p\right)=\dfrac{\varphi_{i,j}}{\lambda_j^i},i=\overline{0,2s+1}, \\ a_{1,j}e^{\lambda_j}+\sum\limits_{p=1}^{s}e^{\lambda_j\alpha_p}\left(b_{p,j}\cos\left(\lambda_j\beta_p+i\theta_p\right)+b_{p+s,j}\sin\left(\lambda_j\beta_p+i\theta_p\right)\right)+ \\ +\sum\limits_{p=s+1}^{2s+1}e^{\lambda_j\alpha_p}\left(c_{p,j}\cos\left(\lambda_j\beta_p+i\theta_p\right)+c_{p+s+1,j}\sin\left(\lambda_j\beta_p+i\theta_p\right)\right)=\dfrac{\varphi_{i+2s+2,j}}{\lambda_j^i},i=\overline{0,2s}. \end{cases} \tag{13}$$

The main matrix of system (13) has the form

$$\Delta_{(4s+3)\times(4s+3)}=\begin{pmatrix} A_{(2s+2)\times(2s+1)} & B_{(2s+2)\times(2s+2)} \\ C_{(2s+1)\times(2s+1)} & D_{(2s+1)\times(2s+2)} \end{pmatrix},$$

where

$$A=$$

$$=\begin{pmatrix} 1 & 1 & . & 1 & 0 & . & 0 \\ 1 & \cos\theta_1 & . & \cos\theta_s & \sin\theta_1 & . & \sin\theta_s \\ 1 & \cos 2\theta_1 & . & \cos 2\theta_s & \sin 2\theta_1 & . & \sin 2\theta_s \\ . & . & . & . & . & . & . \\ 1 & \cos\left(2s+1\right)\theta_1 & . & \cos\left(2s+1\right)\theta_s & \sin\left(2s+1\right)\theta_1 & . & \sin\left(2s+1\right)\theta_s \end{pmatrix},$$

$$B=\begin{pmatrix} 1 & . & 1 & 0 & . & 0 \\ \cos\theta_{s+1} & . & \cos\theta_{2s+1} & \sin\theta_{s+1} & . & \sin\theta_{2s+1} \\ \cos 2\theta_{s+1} & . & \cos 2\theta_{2s+1} & \sin 2\theta_{s+1} & . & \sin 2\theta_{2s+1} \\ . & . & . & . & . & . \\ \cos\left(2s+1\right)\theta_{s+1} & . & \cos\left(2s+1\right)\theta_{2s+1} & \sin\left(2s+1\right)\theta_{s+1} & . & \sin\left(2s+1\right)\theta_{2s+1} \end{pmatrix},$$

$$C=\begin{pmatrix} C_1 & C_2 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} e^{\lambda_j} & e^{\lambda_j \alpha_1} \cos \lambda_j \beta_1 & . & e^{\lambda_j \alpha_s} \cos \lambda_j \beta_s \\ e^{\lambda_j} & e^{\lambda_j \alpha_1} \cos\left(\lambda_j \beta_1 + \theta_1\right) & . & e^{\lambda_j \alpha_s} \cos\left(\lambda_j \beta_s + \theta_s\right) \\ . & . & . & . \\ e^{\lambda_j} & e^{\lambda_j \alpha_1} \cos\left(\lambda_j \beta_1 + 2s\theta_1\right) & . & e^{\lambda_j \alpha_s} \cos\left(\lambda_j \beta_s + 2s\theta_s\right) \end{pmatrix},$$

$$C_2 = \begin{pmatrix} e^{\lambda_j \alpha_1} \sin \lambda_j \beta_1 & . & e^{\lambda_j \alpha_s} \sin \lambda_j \beta_s \\ e^{\lambda_j \alpha_1} \sin\left(\lambda_j \beta_1 + \theta_1\right) & . & e^{\lambda_j \alpha_s} \sin\left(\lambda_j \beta_s + \theta_s\right) \\ . & . & . \\ e^{\lambda_j \alpha_1} \sin\left(\lambda_j \beta_1 + 2s\theta_1\right) & . & e^{\lambda_j \alpha_s} \sin\left(\lambda_j \beta_s + 2s\theta_s\right) \end{pmatrix},$$

$$D = \left(D_1 \quad D_2\right),$$

$$D_1 = \begin{pmatrix} e^{\lambda_j \alpha_{s+1}} \cos \lambda_j \beta_{s+1} & . & e^{\lambda_j \alpha_{2s+1}} \cos \lambda_j \beta_{2s+1} \\ e^{\lambda_j \alpha_{s+1}} \cos\left(\lambda_j \beta_{s+1} + \theta_{s+1}\right) & . & e^{\lambda_j \alpha_{2s+1}} \cos\left(\lambda_j \beta_{2s+1} + \theta_{2s+1}\right) \\ . & . & . \\ e^{\lambda_j \alpha_{s+1}} \cos\left(\lambda_j \beta_{s+1} + 2s\theta_{s+1}\right) & . & e^{\lambda_j \alpha_{2s+1}} \cos\left(\lambda_j \beta_{2s+1} + 2s\theta_{2s+1}\right) \end{pmatrix},$$

$$D_2 = \begin{pmatrix} e^{\lambda_j \alpha_{s+1}} \sin \lambda_j \beta_{s+1} & . & e^{\lambda_j \alpha_{2s+1}} \sin \lambda_j \beta_{2s+1} \\ e^{\lambda_j \alpha_{s+1}} \sin\left(\lambda_j \beta_{s+1} + \theta_{s+1}\right) & . & e^{\lambda_j \alpha_{2s+1}} \sin\left(\lambda_j \beta_{2s+1} + \theta_{2s+1}\right) \\ . & . & . \\ e^{\lambda_j \alpha_{s+1}} \sin\left(\lambda_j \beta_{s+1} + 2s\theta_{s+1}\right) & . & e^{\lambda_j \alpha_{2s+1}} \sin\left(\lambda_j \beta_{2s+1} + 2s\theta_{2s+1}\right) \end{pmatrix}.$$

Due to the uniqueness of the problem A, the determinant of system (13) is different from zero. Let's find the largest degree of the exponent when calculating the determinant $\Delta.$ Since all the exponents in the matrix $C_{(2s+1)\times(2s+1)}$ have positive degrees, then, obviously, the largest degree of the exponent is obtained when calculating the product of the determinants

$$\left|C_{(2s+1)\times(2s+1)}\right| \left|B_{(2s+2)\times(2s+2)}\right|.$$

Let's calculate each determinant. Using the Euler formula $e^{iz} = \cos z + i \sin z,$, we have

$$\det C = e^{\lambda_j \left(1 + 2\sum\limits_{p=1}^{s} \alpha_p\right)} \left(\frac{i}{2}\right)^s \begin{vmatrix} 1 & 1 & . & 1 & 1 & . & 1 \\ 1 & e^{i\theta_1} & . & e^{i\theta_s} & e^{-i\theta_1} & . & e^{-i\theta_s} \\ . & . & . & . & . & . & . \\ 1 & e^{2si\theta_1} & . & e^{2si\theta_1} & e^{-2si\theta_1} & . & e^{-2si\theta_s} \end{vmatrix}.$$

The last determinant is the Vandermonde determinant, which is calculated explicitly

$$\det C = 4^s e^{\lambda_j \left(1 + 2\sum\limits_{p=1}^{s} \alpha_p\right)} \prod_{r=1}^{s} \sin \theta_r \prod_{r=1}^{s} \sin^2 \frac{\theta_r}{2} \prod_{r>t} 4 \sin^2 \frac{\theta_r - \theta_t}{2} \prod_{r \neq t} (-4) \sin^2 \frac{\theta_r + \theta_t}{2}. \qquad (14)$$

Since in expression (14) each factor is different from zero, then $\det C \neq 0.$ Now we calculate the determinant of the matrix Again using the Euler formula, we have

$$\det B=\left(\frac{i}{2}\right)^{s+1}\begin{vmatrix} 1 & . & 1 & 1 & . & 1 \\ e^{i\theta_{s+1}} & . & e^{i\theta_{2s+1}} & e^{-i\theta_{s+1}} & . & e^{-i\theta_{2s+1}} \\ . & . & . & . & . & . \\ e^{i(2s+1)\theta_{s+1}} & . & e^{i(2s+1)\theta_{2s+1}} & e^{-i(2s+1)\theta_{s+1}} & . & e^{-i(2s+1)\theta_{2s+1}} \end{vmatrix}.$$

We got the Vandermonde determinant again. Having performed calculations and some transformations, we obtain

$$\det B=\prod_{r=s+1}^{2s+1}\sin\theta_r\prod_{r>t}4\sin^2\frac{\theta_r-\theta_t}{2}\prod_{r\neq t}(-4)\sin^2\frac{\theta_r+\theta_t}{2}\neq 0.$$

So,

$$\det\Delta=e^{\lambda_j\left(1+2\sum_{p=1}^{s}\alpha_p\right)}N+f\left(\lambda_j\right), \tag{15}$$

where

$$N=4^s\prod_{r=1}^{2s+1}\sin\theta_r\prod_{r=1}^{s}\sin^2\frac{\theta_r}{2}\prod_{r>t\geq 1}^{2s+1}4\sin^2\frac{\theta_r-\theta_t}{2}\prod_{\substack{r\neq t\\ r,t=1}}^{2s+1}(-4)\sin^2\frac{\theta_r+\theta_t}{2}\neq 0.$$

$$f\left(\lambda_j\right)=o\left(e^{\lambda_j\left(1+2\sum_{p=1}^{s}\alpha_p\right)}\right),\lambda_j\to+\infty.$$

To solve problem (11), we have the following estimate:

$$\left|X_j(x)\right|\leq\left|a_{1,j}\right|e^{\lambda_j x}+\sum_{p=1}^{s}e^{\lambda_j\alpha_p x}\left(\left|b_{p,j}\right|+\left|b_{p+s,j}\right|\right)+\sum_{p=s+1}^{2s+1}\left(\left|c_{p,j}\right|+\left|c_{p+s+1,j}\right|\right).$$

Let us now estimate the coefficients

$$\left|a_{1,j}\right|=\frac{\left|\det\Delta^1\right|}{\left|\det\Delta\right|},\left|b_{p,j}\right|=\frac{\left|\det\Delta^{p+1}\right|}{\left|\det\Delta\right|},p=\overline{1,2s}.$$

$$\left|c_{p,j}\right|=\frac{\left|\det\Delta^{p+s+1}\right|}{\left|\det\Delta\right|},p=\overline{s+1,3s+2}.$$

where Δ^r is the matrix obtained from the main matrix of the system by replacing the r th column with the right side of the system (13). Using the Laplace formula, we expand $\det\Delta^r$ in its r th column:

$$\det\Delta^r=\sum_{i=0}^{4s+2}\frac{\varphi_{i,j}}{\lambda_j^i}\Delta_{ri+1}^r,$$

where Δ_{ri+1}^r are algebraic additions. From this we obtain the estimates

$$\left|\det\Delta^1\right|\leq K_1e^{\lambda_j\left(1+2\sum_{i=1}^{s}\alpha_i\right)-\lambda_j}\sum_{i=0}^{4s+2}\left|\varphi_{i,j}\right|,$$

$$\left|\det\Delta^{p+1}\right|\leq K_{p+1}e^{\lambda_j\left(1+2\sum_{i=1}^{s}\alpha_i\right)-\lambda_j\alpha_p}\sum_{i=0}^{4s+2}\left|\varphi_{i,j}\right|,p=\overline{1,s},$$

$$\left|\det\Delta^{p+s+1}\right| \le K_{p+s+1} e^{\lambda_j\left(1+2\sum_{i=1}^{s}\alpha_i\right)-\lambda_j\alpha_p} \sum_{i=0}^{4s+2}\left|\varphi_{i,j}\right|, \; p=\overline{1,s},$$

$$\left|\det\Delta^{p+s+1}\right| \le M_p e^{\lambda_j\left(1+2\sum_{i=1}^{s}\alpha_i\right)} \sum_{i=0}^{4s+2}\left|\varphi_{i,j}\right|, \; p=\overline{s+1,2s+1},$$

$$\left|\det\Delta^{p+2s+2}\right| \le M_{p+s+1} e^{\lambda_j\left(1+2\sum_{i=1}^{s}\alpha_i\right)} \sum_{i=0}^{4s+2}\left|\varphi_{i,j}\right|, \; p=\overline{s+1,2s+1},$$

where $K_1, K_{p+1}, K_{p+s+1}, M_p, M_{p+s+1}$ are some positive constants. Taking into account the last five estimates and (15), we finally obtain the main estimate:

$$\left|X_j(x)\right| \le M\sum_{i=0}^{2n}\left|\varphi_{i,j}\right|, \tag{16}$$

2. For $n=2s$, the same reasoning and calculations will again lead us to estimate (16).

The following theorem is true.

Theorem 2. If the boundary functions $\varphi_i(y), i=\overline{0,2n},$ satisfy the following conditions:

1) $\varphi_i(y)\in C^{4s}[0,1],$

2) $\varphi_i^{(j)}(1)=0,\ j=\overline{0,3s-1},$

3) $\varphi_i^{(j)}(y)=O\left(y^{\alpha-j}\right),\ j=\overline{0,4s}, \alpha>4s-\dfrac{3m+1}{2},$

then the solution to problem A exists in the form of a series

$$u(x,y)=\sum_{j=1}^{\infty}X_j(x)Y_j(y), \tag{17}$$

where $Y_j(y), X_j(x)$ are the solutions of problems (6), (11), respectively.

Theorem 2 is proved by using inequalities (9), Cauchy-Bunyakovsky and Bessel.

The uniqueness of the solution of the formulated problem follows from the completeness of the system of eigenfunctions of problem (6) in the space L_2.

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